Exact Power Law Metrics, Qualitative Analysis of Homogeneous Minisuperspace Dynamics and the Geometry of Minisuperspace

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The scale invariance of the perfect fluid Einstein equations, namely invariance under constant scale transformations of the unit of length, is rather useless in the infinite dimensional case of spacetimes without symmetry. Conformal transformations, on the other hand, play an important role in isolating the dynamical degrees of freedom of the gravitational field.[1] In the finite dimensional case of spacetimes constructed from homogeneous spatial metrics, these two distinct actions coincide and lead to the decoupling of the pure scale (or conformal) degree of freedom from the scale invariant (or conformally invariant) variables.[2] Using the Hamiltonian approach,[2,3] one can then obtain a reduced Hamiltonian system for the latter variables which can be studied using the qualitative theory of differential equations. The reduced system is easily compactified using the fact that the super-Hamiltonian is positive definite in the scale invariant momenta of the reduced system and the scale invariant variables which appear in the spatial curvature, except in a neighborhood of isotropy in the Bianchi type IX case. The singular points of this system determine the qualitative behavior of anisotropic cosmological models both at early times near the big bang and at late times after the big bang. Those singular points in the physical region (nonnegative energy, nondegenerate metric) correspond to exact power law metrics, metrics which admit a transitive spacetime similarity group.[4,5]

The geometry of the Einstein equations is well known[6] and especially relevant in the finite dimensional setting[7] where ordinary differential equations allow it to be very usefully exploited. The evolution of the spatial metric in the space of metrics is deflected from geodesic motion with respect to the DeWitt metric by the spatial curvature and source potentials. By using variables on the finite dimensional space of metrics which are adapted to the spatial gauge symmetry,[4] one is naturally led to employ a particular orthonormal frame on this space. Singular point solution curves have momenta which are conformally related to constants (the scale invariant momenta) and along which the structure functions of the frame (which determine the connection coefficients which appear in the Einstein equations) are constants; those scale invariant quantities on which the spatial curvature essentially depends must also be constant. These conditions immediately imply the exact power law conditions which guarantee that the solution be an exact power law solution.

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