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Electromagnetic self-mass of the classical electron. An alternative exploitation of Fermi's claim for rigid motion

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Abstract

It is demonstrated how the notorious factor of 4/3, occurring in the electromagnetic mass of the classical electron, can be eliminated by a redefinition of the concept of rigid accelerated motion. The mass derived from the self-force on the electron is thus reconciled with Einstein's mass–energy relation. © 1997 Elsevier Science B.V.

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1. Introduction

In the classical model, introduced by Abraham and Lorentz at the beginning of the century, the electron is pictured as a rigid spherical symmetric charge distribution of total charge e. The contribution to the electron mass from its electromagnetic field can in principle be found in two different ways. With the electron moving at constant speed v_x, one can calculate the associated field momentum P_x. The mass then follows as

$$m_p = P_x / v_x . \tag{1}$$

Alternatively, if the electron is accelerated with a constant linear acceleration a_x, an electromagnetic self-force F_x opposing the change in state of motion will result. The corresponding inertial mass is

$$m_s = -F_x / a_x . \tag{2}$$

(If the motion is relativistic, powers of the Lorentz factor will enter.)

Early "naive" calculations gave as a result

$$m_p = m_s = 4U/3c^2 , \tag{3}$$

where U is the total electrostatic energy of the electron and c the velocity of light.

Contrary to expectations, these results do not conform with Einstein's mass–energy relation, and the unwanted factor of 4/3 has been under debate since the days of Abraham and Lorentz.

A solution to this apparent inconsistency for the self-mass was first given by Fermi [1] as early as in 1922 by a covariant application of Hamilton's principle. Fermi's explanation seemingly went unnoticed, but his basic idea of the necessity for covariance was later rediscovered repeatedly by other authors in connection with the momentum derived mass m_p. The procedure for eliminating the notorious factor of 4/3 is essentially to redefine the electromagnetic stress tensor as described by Rohrlich [2,3] and by Jackson [4].

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In contrast, very little is found in the literature (since Fermi) about the inconsistency of the mass m_s derived from the force on the electron on itself. We here demonstrate how a slight modification of the standard textbook procedure harmonizes the self-mass m_s with relativity.

2. Rigid motion

The following phrase is translated from Fermi's 1922 article: "The difference between the two values [U/c^2 and $(4U/3c^2)$] is due to the implicit application, in ordinary electromagnetic theory, of the non-permitted concept of rigid bodies."

In the Lorentz model, the charge distribution representing the electron is assumed to move rigidly in the sense that for a given instant of time, all charge elements have the same *velocity* and *acceleration*. However, if we assume that no elastic stresses are set up, the extended electron must be Lorentz contracted along the direction of motion when observed in the laboratory frame. Furthermore, accelerated motion means a time dependent progressive contraction which in turn implies that charge elements at the "rear" end must be accelerated more than elements at the "front" end.

A rigid accelerated motion of the electron can then be defined as a motion during which the electron shrinks in the direction of motion with the instantaneous Lorentz factor relative to the laboratory frame [5]. This condition is fulfilled only when each point of the extended electron performs a hyperbolic motion with proper acceleration [6].

$$a=\alpha\left(1+\alpha\bar{x}/c^2\right)^{-1}. \tag{4}$$

Here $\bar{x}$ is the coordinate along the direction of acceleration in the co-accelerating frame and α the proper acceleration of the origin $\bar{x}=0$, here taken to be the centre of the electron.

3. The self-field

Standard presentations of the self-force in the classical electron theory are given in the textbooks by Heitler [7] and by Panofsky and Phillips [8]. The charge distribution representing the electron, accelerated in the x-direction, is assumed to be instantaneously at rest at time $t=0$ with its centre coinciding with the origin of the laboratory frame.

The electric field felt by charge element $\mathrm{d}e$ with position r due to another element $\mathrm{d}e'$ at r', is found by a series expansion of the retarded Lienard–Wiechert field to be

$$\mathrm{d}E=\mathrm{d}e'\left(-\frac{2Xa'}{c^2R^3}+\frac{1}{R^3}\right)R, \tag{5}$$

where $R=r-r'$ and X is the component of R along the direction of acceleration. All quantities refer to the time of observation $t=0$. If the typical dimension of the charge distribution is of the order of r_0, the retarded times involved are of the order of r_0/c, corresponding to retarded electron velocities of the order of $\alpha r_0/c$. Relativistic effects can thus be neglected when

$$\alpha r_0/c^2 \ll 1, \tag{6}$$

a condition fulfilled for most reasonable accelerations.

4. The self-mass

The force on element $\mathrm{d}e$ is simply $\mathrm{d}e\,\mathrm{d}E$. Adding up contributions from all pairs of charge elements, only the x-component will be different from zero due to symmetry.

The contribution to the self mass from the $\mathrm{d}e$– $\mathrm{d}e'$ interaction is thus:

$$\mathrm{d}m=-\mathrm{d}e\,\mathrm{d}E_x a^{-1}=\mathrm{d}e\,\mathrm{d}e'\left(\frac{2X^2a'}{c^2R^3}-\frac{X}{R^3}\right)a^{-1}. \tag{7}$$

Note that a' and a are the laboratory frame accelerations of $\mathrm{d}e'$ and $\mathrm{d}e$ respectively. In the standard treatments these quantities are assumed equal, but our modified definition of rigidity makes a' and a depend on x' and x respectively.

Since the electron was assumed instantaneously at rest at $t=0$, the coordinates and accelerations in the laboratory frame and the co-accelerating frame are the same. From Eq. (4) then follows that

$$a=\alpha\left(1+\alpha x/c^2\right)^{-1}, \tag{8}$$

with a similar expression for a'. The condition (6) shows that a differs little from α throughout the charge distribution.

Disregarding terms with positive powers of α, Eqs. (7) and (8) give the total self-mass as

$$m_s = \int \mathrm{d}e\,\mathrm{d}e' \left(\frac{2X^2}{c^2R^3} - \frac{Xx}{c^2R^3} - \frac{X}{\alpha R^3} \right), \tag{9}$$

where the integration is to be carried out over all pairs of charge elements. Note that to this order in α, the additional term in Eq. (9) compared to Eq. (7) stems from the x-dependence of a only. The acceleration a' of the source element can be replaced by α.

The last term in the parenthesis corresponds to the instantaneous Coulomb interaction and gives no contribution upon integrating. Making use of the assumed spherical symmetry in the electron's charge distribution, the integrals of Eq. (9) can easily be expressed as

$$\int \mathrm{d}e\,\mathrm{d}e'(2X^2/R^3) = \tfrac{4}{3}U,$$

$$\int \mathrm{d}e\,\mathrm{d}e'(Xx/R^3) = \tfrac{1}{3}U, \tag{10}$$

where U is the total electrostatic energy stored in the electron

$$U = \int \frac{\mathrm{d}e\,\mathrm{d}e'}{2R}. \tag{11}$$

Our final result is thus

$$m_s = U/c^2, \tag{12}$$

in accordance with expectations.

5. Conclusion

The modification of the self-mass calculation presented above is based on a redefinition of the concept of rigid motion. This introduces the extra factor of

$$a^{-1} = (1 + \alpha x/c^2)\alpha^{-1} \tag{13}$$

in the mass element $\mathrm{d}m$ of Eq. (7).

Formally, the nominator, being the square root of the metric tensor component g_{00} in the accelerated frame, can be absorbed in the electric field of Eq. (5) by the substitution

$$\mathrm{d}E \to \mathrm{d}E(1 + \alpha x/c^2), \tag{14}$$

where α now must be considered as a common acceleration throughout the system. This, in turn, can be interpreted to mean that a rigidly accelerated frame behaves like an anisotropic dielectric medium with a permittivity [6,9]:

$$\varepsilon = g_{00}^{-1/2} = (1 + \alpha x/c^2)^{-1}. \tag{15}$$

The modification (14) effectively introduces an additional term in the electric field of Eq. (5):

$$\delta E = \mathrm{d}e'(R/R^3)(\alpha x/c^2), \tag{16}$$

in agreement with Fermi's findings [1].

With this additional field, the total self-force is readily found to be

$$F_x = -(U/c^2)\alpha. \tag{17}$$

The corresponding self-mass, now defined as F_x/α, is again given by Eq. (12).

We have been concerned here only with the consistency of the electromagnetic features of the classical electron and we did not touch upon the delicate and controversial question of the need for stabilizing Poincaré forces [10–12]. After quantum electrodynamics, this aspect seems to be of historical interest only.

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