E.2 Dino Boccaletti: When a problem is solved too early. Enrico Fermi and the infamous 4/3 problem

Introduction

It has often happened, particularly in the past centuries, that some scientific results had been reobtained more than once, each time ignoring the authors of the preceding discoveries. In the case of mechanics this happened many times, as recalled by A. Wintner in the preface to his famous book:[1] "... even the classical literature of the great century of celestial mechanics appears to be saturated with rediscoveries (sometimes *bona fide* and sometimes not assuredly so) ...". In times closer to us, this has happened again for "the infamous 4/3 problem."[2] It took thirty years for the result obtained by Fermi to have its "consecration" in an authoritative book (see below) and ten more to begin circulating among the community of experts. In the next pages we shall first try to historically contextualize Fermi's paper in an extremely concise way and then to bring into question the procedures through which the paper itself has been interpreted. At the end we shall advance a conjecture which, as with all conjectures, is based on circumstantial but not incontrovertible evidence.

The story in short

In the early twenties, when Fermi was concluding his studies at the University of Pisa, in Italy the problems related to the rising quantum mechanics had not yet filtered into academic circles. Instead the electromagnetic theory and the theory of relativity (special and general) were well-known and studied (even if in restricted circles) through the works of Abraham, Lorentz, Poincaré, Richardson . . . for electromagnetic theory and the papers of Einstein, Levi-Civita, and the book of Weyl for the theory of relativity. Fermi, still a student, had a deep knowledge of these theories and of classical analytical mechanics. Besides being testified to in Fermi's biography written by Emilio Segrè,[3] this appears clearly in the first papers he published.[4] Paper 1) is substantially a generalization of a result which, at that time, was quoted in the circulating textbooks on electrodynamics.

Besides the various editions of the Abraham's *Theorie der Elektrizität* (which originated as a second part of the treatise of Föppl published for the first time in 1894) and Lorentz's *The theory of electrons* (1909, second edition 1915), the textbook having a larger circulation was Richardson's *The electron theory of matter* (1914). Fermi refers to this latter textbook. At that time (1921–23) it was generally accepted that, for a charged particle moving with variable velocity,

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[1] A. Wintner: *The Analytical Foundations of Celestial Mechanics*, Princeton University Press, 1947, p. IX.

[2] The expression is due to J. D. Jackson in his textbook *Classical Electrodynamics*, Third Edition, Wiley 1998, p. 755.

[3] E. Segrè: *Enrico Fermi Physicist*, The University of Chicago Press, 1970

[4] For the first few Fermi's papers also see, in Italian,
C. Tarsitani: *I lavori di Fermi sulla relatività nei commenti di Persico e Segrè*, Atti del IV congresso nazionale di storia della fisica, Como, 1983,
F. Cordella, F. Sebastiani: *Il debutto di Enrico Fermi come fisico teorico: I primi lavori sulla relatività (1921–1922–23)*, Quaderno di Storia della Fisica N. 5, 1999 and
F. Cordella, A. De Gregorio, F. Sebastiani: *Enrico Fermi: Gli anni italiani*, Editori Riuniti, 2001.
To avoid possible misunderstandings, we follow the convention of the present volume and refer to Fermi's papers making use of the numbered classification scheme given in *Enrico Fermi: Note e Memorie (Collected Papers)*, Accademia Nazionale dei Lincei and University of Chicago Press, Vol. 1, 1961, Vol. 2, 1965. The relevant papers 1), 2), 3), 4c) of which we will be concerned in the following are given in English translation in Chapter 2 of this volume.

the electromagnetic mass was 4/3 times the inertial mass.[5] The whole theoretical work done in the last two decades, mainly by Abraham and Lorentz, had led to considering the electron (discovered by J. J. Thomson in 1897) to be a rigid sphere with a uniform charge distribution on its surface. In particular, Abraham was convinced that the electron's entire mass was of electromagnetic origin and in 1902 announced the realization of an "electromagnetic mechanics." He also called "longitudinal mass" the mass associated only with a force oriented along the electron's trajectory and called "transverse mass" that associated with a force oriented perpendicular to the electron's trajectory.[6] (These terms had a long life since were used in various papers on the special theory of relativity, including the fundamental Einstein paper of 1905). Since $E_0^e = \frac{e^2}{2R}$ (R radius of the sphere) is the electrostatic energy, the current theory drove to evaluate the electromagnetic contribution to the electron's mass as $m_e = \frac{2}{3}\frac{e^2}{c^2 R}$. As a consequence this made the electromagnetic mass equal to 4/3 times the mass entering into Einstein's equation $E = mc^2$.

Fermi demonstrates that, in the context of the then current theory, one obtains the same result for any system of moving charges, i.e., the factor 4/3. Therefore inertial mass and electromagnetic mass do not match. He also announces that in a forthcoming paper he will consider electromagnetic masses as masses endowed with weight from the point of view of the general theory of relativity. In point of fact, in paper 2), Fermi obtains the result that the electromagnetic mass and the passive gravitational mass (the weight of the charged particle) do match. This is a blatant contradiction: either this result disproves the equivalence principle (largely accepted by that time) or a new problem arises on the possible electromagnetic nature of mass (remember Abraham's ideas on "electromagnetic mechanics"!). In paper 4c), Fermi first solves the problem. He is well aware of the importance of the result obtained. In fact he writes and publishes three equivalent versions of his work (in *Il nuovo Cimento*, in the *Rendiconti dell'Accademia Nazionale dei Lincei* and in *Physikalische Zeitschrift*).[7] He also confides to his friend Enrico Persico that there will be some troubles to obtain an agreement with his ideas: "*... I am trying with great effort to launch the business of the 4/3. The main difficulty derives from the fact that they have a hard time understanding—in part because the thing is not easy to understand, in part because I express myself too concisely—but little by little they begin to understand what it is all about ...*".[8]

But, as the saying goes, no man is a prophet in his own country and the three versions (even the German one) went unnoticed. Thus, as Rohrlich said,[9] the result was bound to be rediscovered. It did not find its way into the standard references or textbooks until 1953 when E. T. Whittaker, in the second volume of his *History* (on p. 51, see footnote 6) quoted Fermi's Lincei communications saying "It was shown long afterwards by E. Fermi that the transport of the stress system set up in the material of the sphere should be taken into account, and that when this is done, Thomson's result becomes

$$\textit{Additional mass} = \frac{1}{c^2}\ \textit{Energy of the field}\text{ ''}$$

In the meantime two papers had appeared. W. Wilson obtained the same result of Fermi in a different way[10] and analogously B. Kwal 13 years later in a short note arrived at the same

[5]See, for instance O. W. Richardson: *The electron theory of matter*, Cambridge University Press, 1914, Chapters XI, XII.

[6]For a historical analysis of the problem of the electromagnetic mass see
A. I. Miller: *Albert Einstein's Special Theory of Relativity, Emergence (1905) and Early Interpretation (1905–1911)*, Springer, 1998 and
E. T. Whittaker: *A History of the Theories of the Aether and Electricity*, Thomas Nelson & Sons, London 1951 and 1953.

[7]Besides the *Nuovo Cimento* version 4c), Fermi published the two Lincei communications XXI, 1922, pp. 184–187 and 306–309 (4a) and the paper *Über eine Widerspruch zwischen der elektrodynamischen und relativistischen theorie der elektromagnetischen Masse* in Physikalische Zeitschrift XXIII, 340-344, 1922 (4b).

[8]E. Segrè, op. cit., p. 197.

[9]F. Rohrlich: *Charged Classical Particles*, Addison-Wesley, 1965, p. 17.

[10]W. Wilson: The mass of a convected field and Einstein's mass-energy law, Proc. Phys. Soc. (London) **48**, 736–740 (1936). This paper is also mentioned in Whittaker's book.

conclusions exploiting the relativistic transformation of the electromagnetic energy-momentum tensor.[11] Finally, the result was discovered for a fourth time by F. Rohrlich,[12] again (apparently) without the knowledge of any of the previous papers. Fundamentally, Fermi showed that factor 4/3 was produced by an incorrect application of (or more precisely by failing to apply) the theory of relativity. The circumstance which, at first sight, might appear rather strange is that Fermi, in his teaching activity of those years continued teaching the old result. Only in his textbook *Introduzione alla Fisica Atomica*[13] he introduced a short sentence mentioning relativistic corrections (without demonstration). In this connection, W. Joffrain[14] put forward the hypothesis of a sort of deontological scruple: not to teach, in an institutional course, results which are not yet universally accepted. Subsequently, in collaboration with A. Pontremoli,[15] Fermi applied successfully the same method to the calculation of the mass of the radiation contained in a cavity with reflecting walls, for which the standard textbooks of the time had an expression containing the same factor 4/3. Anyway, the problem of the nature of the electromagnetic mass was been dragging on for various decades through the contributions, after that of Fermi, of Rohlrich, Dirac, etc.. However, almost always, the successive results—at least apparently—went unnoticed.

The resistible path of Fermi's paper

At this point, in retrospect, if we look at the whole story some circumstances appear at the very least to be strange. Let us start from the beginning of the sequence. Fermi obtained the result published in 4c) in January 1922.[16] It is clear that he feels proud of the conclusions obtained. This turns out clearly in the letter to Persico in which he already announces his intent of publishing the paper also on a German review (which will result to be *Physikalische Zeitschrift*), to make it known outside of Italy.

At this point we can notice that, completely immersed in the academic context of those times, Fermi thought that the paper concerning the factor 4/3 was much more important, since it was solving a problem already several decades old, than paper 3), only published in Italian in *Rendiconti dell'Accademia Nazionale dei Lincei* presented by G. Armellini in January 1922. As we know, paper 3), after the generalization due to Walker (1932), spread far and wide and still is considered of lasting importance. The German version of 4c), i.e., 4b), sent to *Physikalische Zeitschrift*, was received by the journal the ninth of May 1922. The paper was immediately published and also reviewed in *Physikalische Berichte* by Erich Kretschmann in the issue of December 15.[17] We point out that Erich Kretschmann, who was a habitual reviewer of the journal for at least three sections regarding the foundations of physics (in German: *Allgemeines, Allgemeine Grundlagen der Physik, Mechanik*, respectively), was not an obscure physicist, but a quite well known expert in the theory of relativity. In fact a paper published by him in 1917 on the physical meaning of

[11] B. Kwal: Les expressions de l'énergie et de l'impulsion du champ électromagntique propre de l'électron en mouvement, J. Phys. Radium **10**, 103–104 (1949).
[12] F. Rohrlich: Self-energy and stability of the classical electron Am. J. Phys. **28**, 639–643 (1960).
[13] E. Fermi: *Introduzione alla Fisica Atomica*, Zanichelli, 1928, p. 66.
[14] W. Joffrain: *Un inedito di Enrico Fermi — Elettrodinamica*, Atti del XVIII Congresso di Storia della Fisica e dell'Astronomia, Como (Italy), May 15–16, 1998.
[15] E. Fermi, A. Pontremoli: *Sulla massa della radiazione in uno spazio vuoto*, Rend. Lincei, **32** (1), 162–164 (1923).
[16] This date can be fixed at a sufficiently good approximation by comparing the Fermi's letter to Enrico Persico (see note 8), which is of January twenty-five, 1922, with what Persico writes in *Note e Memorie* Vol. 1, p. 24, introducing the paper. Persico also reports a discussion of Fermi with Luigi Puccianti and Giovanni Polvani regarding the factor 4/3 which seems to coincide with what Fermi writes in the letter (where, however, Fermi does not name the names). The strange thing is that Persico does not quote the letter here. Moreover Fermi dates "January 1922" the German version of the paper.
[17] *Physikalische Berichte, Dritter Jahrgang* 1922, N. 24, p. 1293.

the postulates of the theory of relativity[18] had caused a lot of talk and even aroused a reply by Einstein himself.

Then Fermi's paper, which clarified how one can correctly apply the principles of the (special) theory of relativity to solve the problem of the factor 4/3, outwardly appears to have fallen into the hands of the right person. Unfortunately, this was not the case. In fact instead of investigating the method used by Fermi to applying correctly the relativistic concepts, Kretschmann limited himself to repeating Fermi's words describing the two possible ways of performing the variation for applying Hamilton principle and then to conclude that the solution of the problem of the factor 4/3 given by von Laue in his book was "*much more transparent*".[19] We know from Fermi's biography written by Segrè and from the reminiscences published by Persico that Fermi had studied Weyl's textbook[20] thoroughly, which moreover is quoted in the paper itself when Fermi follows Weyl in applying Hamilton's principle. It is enough to make a comparison of Fermi's paper with the page where Weyl says "*This theory does not, of course, explain the existence of the electron, since cohesive forces are lacking in it*"[21] for understanding that Fermi, following Weyl, only means to deal with a charged sphere (with a surface distribution of charge) without tackling the problem of its internal structure and stability. Then the comparison that Kretschmann makes with von Laue's solution, which involves the introduction of the so called "Poincaré stresses" which turn out to be necessary for ensuring the stability of the electron, is completely misleading. Fermi, as those who will find the solution of the factor 4/3 after him, considers this problem as having nothing to do with the problem of stability. It is curious that even Enrico Persico, who in January 1922 received the letter in which Fermi mentioned the subject, in 1961 writes "*It is now well known that the factor 4/3 can be interpreted as due to the part of the energetic tensor contributed by the internal non-electromagnetic stresses, whose existence must be assumed to assure the equilibrium of the charges. However, in the books known to Fermi, this discrepancy was not explained (he had evidently overlooked the explanation contained in M. von Laue, Die Relativittstheorie, 1, third edition, 1929, p. 218) and so he found for it an explanation of his own, essentially equivalent to the former but obtained through Weyl's variational method*".[22] At that date Rohrlich's paper had already been published, but perhaps Persico had not had enough time to see it. However, a good eight years before (1953) the second volume of Whittaker's book[23] had been published in which Fermi's paper (the Lincei version) was mentioned with the explanation reported above. We point out that Whittaker's book did not go unnoticed, both for the reputation of the author and for the *vexata quaestio* of the authorship of the special theory of relativity. As is known, Whittaker ascribed to Poincaré the authorship of the special theory of relativity and was also charged with ahistoricisms concerning the theory of relativity.[24] Then it is strange that even Rohrlich did not know about the quotation of Fermi by Whittaker, particularly if we bear in mind that the subject of Whittaker's book was the origin and the development of the e. m. theory (Abraham, Poincaré, Lorentz, ...). In the 1960 paper (see 12), which is the first Rohrlich dedicated to the problem of the electromagnetic mass of the electron and related questions, Fermi's paper is not mentioned

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[18]E. Kretschmann: Über den physikalischen Sinn der Relativitätstheorie, Annalen der Physik **53**, 575–614 (1917).

[19]See 17 Kretschmann quotes the 1919 third edition of von Laue's book, but the author continued to maintain the same conclusions in the subsequent fourth edition (see Die Relativitätstheorie on Dr. M. von Laue, Braunschweig, 1929, pp. 224–227 and also its French translation).

[20]Fermi always quoted the fourth 1921 edition of H. Weyl: Raum. Zeit. Materie, Springer, Berlin.

[21]This excerpt is from the English translation of the 1921 German edition republished by Dover in 1952 with the title "*Space-Time-Matter*".

[22]See *Note e Memorie* Vol. 1, p. 24. This strange and uncorrected (for what regards "it is now well known...") sentence has been also remarked by Tarsitani, loc. cit. in 4.

[23]See 6.

[24]See G. Holton: On the Origins of the Special Theory of Relativity (1960) in
G. Holton: *Thematic Origins of Scientific Thought. Kepler to Einstein*, Harvard University Press, 1977 and
A.J. Miller: A study of Henri Poincaré's "Sur la Dynamique de l'Electron," Arch. Hist. Exact. Scis. **10**, 207–328 (1973).

and the same for Wilson's and Kwal's papers.

Apparently Rohrlich solved independently the "4/3 problem" without knowing the contributions of his predecessors. Two years later, in a lecture given before the Joseph Henry Society[25] he said "*. . . For a finite electron this was first pointed out by Fermi in 1922. It is closely related to the definition of rigidity in special relativity where the difference in the simultaneity of relatively moving observers plays an essential role. Unfortunately, Fermi's paper was either never understood or soon forgotten*". Rohrlich, at this point, quotes as a reference the German version (see 7) of Fermi's paper but there is no mention of the papers of Wilson and Kwal. In his 1965 book (see 9), he mentions all three authors (Fermi, Wilson, Kwal) who had preceded him. As a matter of fact this is the last time Rohrlich mentions Fermi's contribution. On this subject he has published papers for about forty years but, as one can check considering the most important journals, Fermi's name is no longer mentioned. The odd thing is that, even in the last paper known to us[26] which contains an appendix with the title "*The history and eventual solution of the stability problem (the 4/3 problem)*", Fermi's name does not appear. A prospective reader could only find the reference to Fermi in the bibliographies of the books and the papers quoted.

In the same year (1965) in which Rohrlich's book appeared, the second revised edition of *The Special Theory of Relativity* by J. Aharoni also came out.[27] In the preface the author says that Rohrlich's 1960 paper "*...initiated new interest in the problem and it turned out that actually a similar solution had already been proposed by B. Kwal in 1949 and the same result obtained as for back as 1922 by E. Fermi who used a different method. It can now be stated that the abolition of the 4/3 factor is also implicit in Dirac's paper on the classical theory of the electron (1938). It is difficult to explain why all the earlier papers passed unnoticed. Possibly this was due to Poincaré's idea to link the 4/3 factor with the instability of an electric charge on purely electrostatic forces*". It should be noted that Aharoni cannot have had knowledge from Rohrlich's paper, which he quotes, of the name of Fermi, Wilson and Kwal since in that paper they are not mentioned. Before the two 1965 books, Kwal's name does not appear, while Wilson's name only appears in Whittaker's book together with Fermi's. Evidently, there is a missing link in the chain!

Let us turn again to Aharoni's book which, from our point of view, assumes a particular importance. In fact, Aharoni is the only one, among all who spoke about Fermi's paper, who devoted himself to effectively understand the method Fermi used for applying his relativistic concept of rigidity. He spends about two pages (170–171) to explain and explicitly reconstruct Fermi's calculations omitted by the author who sums up "*...we have manifestly. . .*". Therefore, on the part of Aharoni there is a true appreciation for the work done by the man who first solved the problem. Retrospectively, it comes to mind that to the early readers those calculations might not be so transparent (see the above letter to Persico where Fermi admits to expressing himself "*too concisely*") and also that the course of differential geometry given by Luigi Bianchi about which Fermi speaks in a letter to Persico, came in very useful to him so to consider obvious the calculations and then to omit them.[28] All the known biographies of Fermi report that he went to Göttingen with a fellowship from the Italian Ministry of Public Instruction in the winter 1922–23 to study with the group headed by Max Born and he remained there seven months. ". . . when Fermi arrived at Göttingen, he found several brilliant contemporaries there, among them Werner Heisenberg and Pascual Jordan, two of the brightest luminaries of theoretical physics. Indeed the two had already been recognized for their exceptional abilities, and Born was writing papers in collaboration with them at about the time of Fermi's residence in Göttingen. Unfortunately it seems that Fermi did not become a member of that extraordinary group or interact with them. I

[25]The theory of the electron, Thirty-first Joseph Henry Lecture (read before the Society May 11, 1962).

[26]F. Rohrlich: The dynamics of a charged sphere and the electron, Am. J. Phys. **65**, 1051-1056, (1997).

[27]J. Aharoni: The special Theory of Relativity, Second revised edition, Oxford University Press, 1965 (reprinted by Dover, 1985).

[28]In the letter Fermi writes "I will pass the examination in higher analysis (differential geometry) which is a terrific bore, in which the problem studied are chosen by the sole criterion that they should lack all interest," see Segrè, op. cit. p. 201–202.

do not know the reason for this ... ".[29] "... Born himself was kind and hospitable. But he did not guess that the young man from Rome, for all his apparent self-reliance, was at the very moment going through that stage of life which most young people cannot avoid. Fermi was groping with uncertainty and seeking reassurance. He was hoping for a pat on the back from Professor Max Born ... ".[30] Both the biographers (Emilio Segrè and Fermi's wife) agree in maintaining that Fermi came back to Rome not satisfied with the German experience, somehow disappointed. It is known from other sources that Born and his collaborators thought the best of Fermi and this is born out from the fact that subsequently he was on friendly terms with them. Moreover, the biographers confirm that Fermi's German was certainly good enough to allow easy communication and then not to be excluded. Why then, as far as we know, he did not join the Born's group and went back to Rome disappointed? Both Emilio Segrè and Laura Fermi put forward the hypothesis that the ideas of Born's group at that time appeared to be very concrete, even philosophical, and then not able to catch the interest of Fermi or Fermi himself was not mature enough to get himself to be appreciated in that environment.

Our conjecture is that Fermi had effectively taken his paper to Göttingen to be appreciated, but he did not achieve his aim. When Fermi arrived in Göttingen, the paper on the "4/3 problem" had already been published in German and so readable by Born and the others. It is unthinkable that Fermi, who was so proud of his result, had not exhibited it and asked Born for his opinion. We recall that, what's more, in his paper Fermi quotes a relativistic definition of rigidity due to Born (in a paper of 1909).[31] The most obvious thing to do for a brilliant young physicist, as Fermi was, would have been to display the paper he was proud of to the authoritative professor. To the best of our knowledge, no proof exists even if it is reasonable to suppose that this had happened. The only thing we can say for certain is that Born's book on Relativity theory,[32] which in its second edition of 1921 held the "traditional" point of view of the "4/3 problem", continued to give the same version till to the last edition.[33] The same thing happened for Pauli's famous lectures[34] as if Fermi's paper had never existed. Born and Pauli were not alone in ignoring Fermi's paper and related conclusions; to the list we can add even Feynman.[35] Coming back to Born, from his authobiography[36] it turns out that over the years he continued to think about the problem of the electromagnetic mass of the electron, but there is no connection with Fermi's conclusions which are never mentioned. Our conjecture, for all its worth, is that the disappointment for having not received appreciation embittered Fermi and also deterred him from the subject. Moreover, the problems raised by the new quantum mechanics and statistical theories definitively averted his interest from classical electrodynamics.

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[29]See Segrè, op. cit. p. 33.

[30]Laura Fermi: *Atoms in the Family. My life with Enrico Fermi*, The University of Chicago Press, 1954, p. 31

[31]Max Born: *Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips*, Annalen der Physik IV, **11**, 1–56 (1909).

[32]Max Born: *Die Relativitätstheorie Einsteins und ihre physikalischen Grundlagen*, Springer, Berlin, 1921, p. 157.

[33]Max Born: *Einstein's Theory of Relativity*, revised edition prepared with the collaboration of Günther Leibfried and Walter Biem, Dover, 1962, pp. 207–214 and 278–289.

[34]W. Pauli: *Pauli Lectures on Physics, Vol. 1. Electrodynamics*, MIT Press, 1972 (reprinted by Dover, 2000), p. 151

[35]*The Feynman Lectures on Physics. The Electromagnetic Field*, Addison-Wesley, 1964, Sect. 28–3 and ff.

[36]Max Born: *My Life. Recollection of a Nobel Laureate*, Taylor & Francis Ltd, 1978, Part 2, IV, pp. 254–255