Chapter 1

Introduction

This contains two chapters including translations and reproductions of key papers by Fermi which are related to astrophysics, together with three appendices of some historically relevant papers by other authors and commentary on some of his articles.

1.1 Fermi's Italian Period

Chapter 2 contains the English translation of the papers originally published in Italian during Fermi's Pisa and Rome periods. The most famous of these introducing Fermi coordinates and Fermi transport (implicitly defining what later became known as Fermi-Walker transport, see Appendix B.2) was indeed a detour from Fermi's initial investigation of electromagnetic mass in special and general relativity that seems to have been largely ignored over the past ninety years. Credit for translation of Fermi's articles from Italian to English goes to: Emanuele Alesci for papers 4c) and 5), Donato Bini and Andrea Geralico for papers (1), (2), and (3), Dino Boccaletti for papers (7), (10), (12), (13), (30), (38) and (80a), and Simone Mercuri for paper (43), using the article labeling system from the two-volume set of Fermi's collected works noted in the preface. Robert Jantzen edited these translations for English expression.

This section contains the English translations of a selection of papers from those Fermi published in Italian in the first part of his scientific career. The seminal papers selected are all related to relativity, astronomy and their applications. For a better account of the circumstances under which the papers were written, we also add excerpts of the presentations due to friends and collaborators of Fermi and published in Volume 1 of Fermi's *Note e Memorie*, 1961.

In paper FI 1 *On the dynamics of a rigid system of electric charges in translational motion* (1), Fermi calculates the inertial mass of a spherical distribution of charge with a constant acceleration by considering the reaction of the charge to its own average field. This leads to the formula $mc^2 = (4/3)U$ relating the inertial mass m to the classical electromagnetic energy U of the distribution. This value, in agreement with a calculation of the electromagnetic mass of a spherical homogeneous shell performed by Lorentz, contradicts the formula $mc^2 = U$ that one would expect from the principle of equivalence of mass and energy. Fermi considers the charge distribution at rest in a homogeneous gravitational field equal to the sign-reversed acceleration which appears to be in agreement with the relativistic formula. This topic is further examined in the subsequent article.

In paper FI 2 *On the electrostatics of a homogeneous gravitational field and on the weight of electromagnetic masses* (2), Fermi reconsiders the calculation of the inertial mass of a spherical distribution of charge using for the first time general relativity, employing a Levi-Civita metric to describe a homogeneous gravitational field in the linear approximation. This approach has been expanded to what is now called the Rindler metric.[1] His final result leads to the desired relation $mc^2 = U$. Another result derived in this paper is the value of the polarization of an infinitesimal conducting sphere at rest in a static gravitational field. An article by R. Ruffini[2] (see Appendix A.5) discusses some general relativistic developments that have taken place in the intervening years for describing electric charges in strong gravitational fields.

Paper FI 3 *On phenomena occurring close to a world Line* (3) is a classic result obtained by Fermi within the framework of general relativity expressing a system of space-time coordinates particularly suited to follow the behavior in time of phenomena happening in a small spatial region around the world line of a particle. Fermi explores the definition of the related coordinate transport which underlies it, later known as "Fermi transport," expressing the metric in the linear approximation for a general space-time. He also expresses Maxwell's equations in these coordinates, supporting the conclusions reached in the previous article.

The contribution by D. Bini and R. Jantzen (B.2) in Appendix B of this volume gives a summary of what we now call Fermi coordinates and Fermi transport with a historical update including Walker's contribution which led to the terminology of "Fermi-Walker transport." This article also discusses the geometry of the various relativistic contributions to the Fermi-Walker transport of vectors around circular orbits in black hole spacetimes and in their Minkowski limit.

In paper FI 4 *Correction of a contradiction between electrodynamics and the relativistic theory of electromagnetic mass* (4c), Fermi reconsiders the problem of the electromagnetic contribution to the mass of an elementary particle already discussed in the previous three articles. The discrepancy between the value $(4/3)(U/c^2)$, obtained by Lorentz for the inertial mass of a rigid, spherically symmetrical system of electric charges, and the value U/c^2 predicted by relativity was well known to Fermi from the previous articles. Such a discrepancy had been interpreted by Poincaré as due to the part of the stress-energy tensor contributed by internal non-electromagnetic stresses, whose existence was assumed to assure the equilibrium

[1] See W. Rindler: *Essential relativity; special, general, and cosmological*, Van Nostrand Reinhold Co., 1969.

[2] *R. Ruffini: Charges in gravitational field: From Fermi, via Hanni-Ruffini-Wheeler, to the "electric Meissner effect", Nuovo Cimento* **119B**, 785–807, 2004.

of the charged particles. A vast scientific literature of followers of this Poincaré's conjecture exists. Fermi shows that by assuming the accelerated charge distribution to be spherically symmetric in its rest frame instead of the laboratory frame, he obtains the correct inertial mass expected from the equivalence principle. This essentially reintroduces the crucial lapse factor between coordinate and physical components of the electric field which is responsible for the correction, to first order in the acceleration, of the approximation made in all of his "Fermi coordinate" system calculations.

The results obtained by Fermi in this paper went unnoticed and for the most part remain that way today. Some of the crucial Fermi results in this paper and the historical developments of this most unique accident in physics are discussed in Appendix A by D. Bini, A. Geralico, R.T. Jantzen and R. Ruffini (see A.1) and by R.T. Jantzen and R. Ruffini in a brief summary of the key mathematics and their consequences (see A.2), as well as in a historical review by D. Boccaletti (see A.3).[3] Interestingly enough, related considerations were also put forward years later by B. Kwal without mentioning Fermi's work. Appendix A.4 reproduces this 1949 paper.

The paper FI 5 *Masses in the theory of relativity* (5)—a short contribution to a collective volume on the foundations of Einstein's theory of relativity—is evidence of the high reputation enjoyed by the young Fermi (age 22) in the physicists' community. Remarkable appears to be the prophetic premonition of things to come. A very favorable attitude toward Einstein theory by the young Fermi is clear at a time in which the older generation of Italian physicists was skeptical and hostile to relativity as recalled by Emilio Segré in Vol. 1, p. 33 of *Note e Memorie*.[4]

In paper FI 6 *On the mass of radiation in an empty space* (10), written in collaboration with Aldo Pontremoli, Fermi successfully applied the method used in FI 4 (4c) to the calculation of the mass of the radiation contained in a cavity with reflecting walls, for which the standard textbooks had an expression containing the same factor 4/3.

The papers FI 7 *The principle of adiabatics and the systems which do not admit angle coordinates* (12) and FI 8 *Some theorems of analytical mechanics of great importance for quantum theory* (13) are dedicated to the theory of adiabatic invariants. The interest of Fermi in the theory of adiabatic invariants, if we make reference to the published papers, goes from 1923 throughout 1926. As the other theoretical physicists in that period, he was convinced of the fundamental importance of the theory of adiabatic invariants for a rigorous formulation of quantum mechanics. On the other hand, Max Born also shared the same opinion[5] .

[3]Boccaletti's review was written before the publication of the paper "The mass of the particles" by A. Bettini (*Rivista del Nuovo Cimento* **32**, No. 7, 2009, pp. 295–337) where Fermi's priority in first resolving this problem is again noted and continuing ignorance of his result by many outstanding authors is recalled as well (see pp. 302–303).

[4]On this topic see also, e.g., Roberto Maiocchi: *Einstein in Italia—La scienza e la filosofia italiana fra le due guerre—Le Lettere*, Firenze, 1985.

[5]See Max Born: *Vorlesungen über Atommechanik*, Berlin, 1925, pp. 58–67, 109–114. English translation: *The mechanics of the atom*, London, 1927, pp. 52–59, 95–99.

Fermi also devoted a lecture in his university course on theoretical physics [6] to the theory of adiabatic invariants and he gave an elementary exposition of it in his book *Introduzione alla Fisica atomica*.[7] His interest was also awakened in conferences and seminars delivered at the University of Rome and in communications at the *Accademia Nazionale dei Lincei*. It was in those occasions that he sparked the interest of an outstanding listener: Tullio Levi-Civita.[8] The involvement of Levi-Civita was such that, besides giving a rigorous mathematical formulation of the subject,[9] he also promoted astronomical applications of the theory. Those due to his collaborator Giulio Krall turned out to be particularly interesting. We must add that in those years James Jeans was also concerned with astronomical applications of the theory of adiabatic invariants.[10]

The paper FI 9 *A theorem of calculation of probability and some of its applications* (38b) is the second part of a Fermi's habilitation thesis to the "Scuola Normale Superiore" of Pisa (1922). It concerns the application of a theorem of calculation of probability to the dynamics of comets. The significance and the potentialities of this paper are well elucidated in the paper of C. Sigismondi and F. Maiolino (B.8) in Appendix B.

The paper FI 10 *Formation of images with Röntgen rays* (7) derives from a part of the degree thesis of Fermi at the University of Pisa. The thesis of Fermi was the most complete survey of X-rays physics in his time. He can also be considered a forerunner of techniques which are standard today. As Sigismondi and Mastroianni say in their article (B.9), although Fermi's seminal ideas are not among the sources investigated by Riccardo Giacconi and Bruno Rossi (1960) when they proposed a telescope using X-rays, Fermi's thesis was the most complete study of X-ray physics at his time. Fermi used the technique of 'mandrels' to form optical surfaces. He anticipated the technique used for the mirrors of Exosat, Beppo-SAX, Jet-X and XMM-Newton telescopes, which is now a mainstay of optical manufacturing. The paper by Sigismondi and Mastroianni discusses this noteworthy connection. It is appropriate here also to recall the comments of Franco Rasetti in the introduction of this article in Volume 1 of Fermi's Note e Memorie. Since at that time *"he had already published or at least completed several important theoretical papers, it may be asked why he did not present a theoretical thesis. It must be explained that at the time in Italy theoretical physics was not recognized as a discipline to be taught in universities, and a dissertation in that field would have been shocking at least*

[6]See A. De Gregorio, S. Esposito: Teaching theoretical physics: The cases of Enrico Fermi and Ettore Majorana, Am. J. Phys. **75** (9), 781–790 (2007).

[7]Enrico Fermi: Introduzione alla Fisica Atomica, Zanichelli, Bologna, 1928, pp. 155–160.

[8]See P. Nastasi, R. Tazzioli: Tullio Levi-Civita, in Lettera Matematica pristem n. 57–58, Springer 2006

[9]We restrict ourselves to quote the last paper on the subject: T. Levi-Civita, A general survey of the theory of adiabatic invariants, *Journal of Math. and Physics* **13**, pp. 18–40 (1934).

[10]J.H. Jeans: Cosmogonic problems associated with a secular decrease of mass, MNRAS **85** (1) 2 (1924).
J.H. Jeans: The effect of varying mass on a binary system, MNRAS **85** (9) 912 (1925).

to the older members of the faculty. Physicists were essentially experimentalists, and only an experimental dissertation would have passed as physics. The nearest subject to theoretical physics, mechanics, was taught by mathematicians as a field of applied mathematics, with complete disregard for its physical implications. These circumstances explain why such topics as the quantum theory had gained no foothold in Italy: they represented a "no man's land" between physics and mathematics. Fermi was the first in the country to fill the gap." (F. Rasetti, Vol. 1, pp. 55–56).

The paper FI 11 *On the quantization of an ideal monoatomic gas* (30) is the communication (to the Accademia Nazionale dei Lincei) in which Fermi expounds for the first time the statistical theory which will be named after him (together with P.A.M. Dirac). The enormous importance of the Fermi-Dirac statistics in astrophysics is recalled in Section 1.1 of Chapter 1.

In the following we give an excerpt from the presentation of Franco Rasetti *"... the present paper, probably his most famous theoretical contribution, where he formulated the theory of an ideal gas of particles obeying the Pauli exclusion principle, now designated in his honor as "fermion."*

There is conclusive evidence to show that Fermi had been concerned with the problem of the absolute entropy constant at least since January 1924, when he wrote a paper (Fermi 20) on the quantization of systems containing identical particles. He had also been discussing these problems with Rasetti several times in the following year. He told much later to Segré that the division of phase space into finite cells had occupied him very much and that had not Pauli discovered the exclusion principle he might have arrived at it a round-about way from the entropy constant (cfr. No. 20).

As soon as he read Pauli's article on the exclusion principle, he realized that he now possessed all the elements for a theory of the ideal gas which would satisfy the Nerst principle at absolute zero, give the correct Sackur-Tetrode formula for the absolute entropy in the limit of low density and high temperature, and be free of the various arbitrary assumptions that had been necessary to introduce in statistical mechanics in order to derive a correct entropy value. He does not seem to have been greatly influenced by Einstein's theory based on Bose's treatment of the black-body radiation as a photon gas, although he points out the analogy between the two forms of statistics. Apparently it took Fermi but a short time to develop the theory in the detailed and definitive form in which it was published in the German version." (F. Rasetti, Vol. 1, p. 178).

The paper FI 12 *A statistical method for the determination of some properties of the atom* (43) translated here is the first of the papers Fermi devoted to the theory of what is today called the Thomas-Fermi atom. Fermi was unaware of the results previously reached by Thomas and his work went on independently for two years. Of great importance are the applications of the Thomas-Fermi model in astrophysics. He was, for example, quite familiar with the applications of his statistics (with the required relativistic modifications) to the theory of the structure of white dwarf stars: indeed, T.D. Lee, as a graduate student of Fermi, wrote his Ph.D. thesis on the *Hydrogen Content and Energy-Productive Mechanism of White Dwarf Stars* (*Ap. J.* **111**, 625, 1950). As we showed the general relativistic generalization of the Thomas-Fermi atom has recently led to a new theoretical framework to study both white dwarfs and neutron stars.

The paper FI 13 *An attempt at a theory of β rays* (80a), translated here, can be described as the birth certificate of the theory of β-decay and weak interactions. Its importance is hardly questionable today. It is here that the possible existence of a massive neutrino is mentioned. At that time (1933) things were not so easy, as Segré describes:

"Fermi gave the first account of this theory to several of his Roman friends while we were spending the Christmas vacation of 1933 in the Alps. It was in the evening after a full day of skiing; we were all sitting on one bed in a hotel room, and I could hardly keep still in that position, bruised as I was after several falls on icy snow. Fermi was fully aware of the importance of his accomplishment and said that he thought he would be remembered for this paper, his best so far. He sent a letter to Nature *advancing his theory but the editor refused it because he thought it contained speculations that were too remote from physical reality; and instead the paper ("tentative theory of beta rays") was published in Italian and in the* Zeitschrift für Physik*. Fermi never published anything else on this subject, although in 1950 he calculated matrix elements for beta decay as an application of the nuclear shell model."* (Emilio Segré: *Enrico Fermi, Physicist*, The University Chicago Press, 1970, p. 72).

1.2 Fermi's American Period

Chapter 3 reproduces some of Fermi's classic papers from his American period regarding the origin of cosmic rays and the mechanism of their acceleration, the interstellar magnetic field and its importance in astrophysics (in this field, Fermi was a pioneer), and the famous Fermi-Pasta-Ulam paper on nonlinear problems. Paper (240) is an article "The origin of the elements" of Fermi in Italian from Fermi's American period recorded by E. Pancini and translated by Dino Boccaletti, the third of nine lectures delivered in an Italian physics conference held in Rome and Milan in 1949, in response to Gamov's attempt to calculate the relative abundances of elements created in the early hot expanding universe. We also include the Fermi-Turkevich article which follows this same argument. A detailed discussion of the story behind these two papers and their relevance for relativistic cosmology can be found in the companion book *Einstein, Fermi, Heisenberg and the Birth of Relativistic Astrophysics* by Remo Ruffini.

We have selected seven of Fermi's papers from his American period to reproduce here, six of which are relevant to astrophysics. We have also added the famous paper "Study of nonlinear problems." All of these have been quoted and commented on numerous times but we think that in order to have a clearer idea of their ideas, it is better to go back to the original sources. As in the preceding chapter, we also include some excerpts of commentary on those papers from Volume 2 of Fermi's *Note e Memorie*.

The first three papers, FA 1 *On the origin of the cosmic radiation* (237), FA 2 *An hypothesis on the origin of the cosmic radiation* (238), FA 3 *Galactic magnetic fields and the origin of cosmic radiation* (265), tackle the problem of the origin of the cosmic rays formulating the hypothesis of a galactic origin and consider the role of the magnetic field. Comments on these papers can also be found in the Ames paper (B.1) in Appendix B.

As recalled by Anderson, *"Paper No. 237 was a direct outcome of heated disputes with Edward Teller on the origin of the cosmic rays. It was written to counter the view that cosmic rays were principally of solar origin and that they could not extend through all galactic space because of the very large amount of energy which would then be required. Taking up the study of the intergalactic magnetic fields, Fermi was able to find not only a way to account for the presence of the cosmic rays, but also a mechanism for accelerating them to the very high energies observed. He presented these same views on the origin of cosmic rays, though less extensively, in a talk at the Como International Congress on the Physics of Cosmic Rays (paper No. 238)."* (H.L. Anderson, Vol. 2, p. 655)

As Chandrasekhar recalls, *"In the fall of 1948, Edward Teller was advancing the view that cosmic rays are of solar origin. Fermi was want to say—half-jokingly—that this inspired him to take an opposing view and advocate a galactic origin of the cosmic rays."* (S. Chandrasekhar, Vol. 2, p. 924)

It is therefore appropriate to recall here Teller's point of view: *"Fermi mentioned to me his interest in the origin of cosmic rays as early as 1946. Several years before that time he mentioned the subject in some lectures in Chicago. He had the suspicion that magnetic fields could accelerate the cosmic particles. In 1948 Alfvén visited Chicago. He had been interested in electromagnetic phenomena on the cosmic scale for quite some time. At that time I was playing with the idea that cosmic rays might be accelerated in the neighborhood of the sun. I had discussed this question with Alfvén, and he visited us in Chicago in order to carry forward the discussion. During this visit Fermi learned from Alfvén about the probable existence of greatly extended magnetic fields in our galactic system. Since this field would necessarily be dragged along by the moving and ionized interstellar material, Fermi realized that here was an excellent way to obtain the acceleration mechanism for which he was looking. As a result he outlined a method of accelerating cosmic ray particles which serves today as a basis for most discussions on the subject. In his papers published in 1949 (Nos. 237 and 238) he explained most of the observed properties of cosmic rays with one important exception: it follows from his originally proposed mechanism that heavier nuclei will not attain as high velocities as protons do. This is in contradiction with experimental evidence. Fermi returned to this problem in his paper Galactic Magnetic Fields and the Origin of Cosmic Radiation (No. 264). Some details concerning the origin of cosmic rays have not been settled conclusively by Fermi's papers. Another competing theory has been proposed by Stirling Colgate and Montgomery Johnson according to which cosmic rays are produced by shock mechanism in exploding supernovae. The actual origin of cosmic rays continues to remain in doubt."* (E. Teller, Vol. 2, p. 655)

As Anderson recalls *"Fermi's interest in astrophysics was welcomed by the astrophysicists. They asked him to give the Sixth Henry Norris Russell Lecture of the American Astronomical Society. Fermi was quite pleased by this show of regard outside his own field and took the occasion to re-examine his earlier ideas about the origin of the cosmic rays in view of later developments in the knowledge of the strength and behavior of the magnetic fields."* (H.L. Anderson, Vol. 2, p. 970). (See also the introduction to paper No. 237.)

The paper FA 4 *High energy nuclear events* (241) was published in the issue of the *Progress of Theoretical Physics* dedicated to the 15th anniversary of the Yukawa theory and considers a statistical description for pion production. As mentioned by Anderson in the comments to this paper in the collected work of Fermi,[11] the methods developed by Fermi were relatively simple, and moreover were deliberately simplified and therefore, were rather useful for experimentalists at that initial phase of high energy physics. Since pions are also bosons, at high energies when their rest mass can be neglected, the concept of temperature can be introduced and the energy density will be given by Stefan's law. Obtaining the temperature from the total energy within a given volume, the number densities of the produced pions

[11] *Fermi: Note e Memorie (Collected Papers)*, Vol. 2, 1965, p. 789.

and nucleons can then be estimated. The role played by thermalization in this paper has inspired us, even though the mechanism is different, namely astrophysical applications in the study of the spectra of gamma ray bursts (GRBs).

It is appropriate to recall here the comment of Isador Rabi in reaction to this paper as told by Anderson: *"Rabi's comment after hearing Fermi present this paper at an American Physical Society meeting in Chicago is worth recording here. 'If Fermi is right in saying that he can calculate what will happen at very high energies by purely statistical methods, then we will have nothing new to learn in this field.' Rabi should have had nothing to fear. Fermi's theory was greatly oversimplified as he intended it to be, and while it did not give very well the detailed results which were later found, it did serve as a standard against which one could make a first comparison of the experimental results of multiple production to reveal when something non-statistical was going on. In the later literature this made it appear that this theory was always wrong; a point that Fermi didn't enjoy at all. He had always stressed the purpose and limitations of his calculations and referred ironically to his own authority and to those who took his results beyond what he intended them to be."* (H.L. Anderson, Vol. 2, p. 789)

Fermi's theoretical papers rarely had co-authors. Among his few co-authors was Chandrasekhar, on two papers on magnetohydrodynamics, FA 5 *S. Chandrasekhar, E. Fermi: Magnetic Fields in Spiral Arms* (261) - FA 6 *S. Chandrasekhar, E. Fermi: Problems of Gravitational Stability in the Presence of a Magnetic Field* (262). Chandrasekhar's recollections on their joint work with remarkable details on Fermi's style of work are published in Volume 2 of Note e Memorie.[12] We give below some excerpts from them. D. Boccaletti comments on the two papers in an article (A.3) of Appendix A.

On paper (262) Chandrasekhar recalls: *"As I have already stated, Fermi and I discussed astrophysical problems regularly during 1952–53. The paper Problems of Gravitational Stability in the Presence of a Magnetic Field (No. 262) was an outcome of these discussions. Referring to this largely mathematical paper, several persons have remarked that it is "out of character" with Fermi. For this reason I may state that the problems which are considered in this paper were largely at Fermi's suggestion. The generalization of the virial theorem; the existence of an upper limit to the magnetic energy of a configuration in equilibrium under its own gravitation; the distortion of the spherical shape of a body in gravitational equilibrium by internal magnetic fields; the stabilization of the spiral arms of a galaxy by axial magnetic fields; all these were Fermi's ideas, novel at the time. But they had to be proved; for, as Fermi said: "It is so very easy to make mistakes in magneto-hydrodynamics that one should not believe in a result obtained after a long and complicated mathematical derivation if one cannot understand its physical origin; in the same way, one cannot also believe in a long and complicated piece of physical reasoning if one cannot demonstrate it mathematically." If only this dictum were followed by all!"* (S.

[12] *Fermi: Note e Memorie (Collected Papers)*, Vol. 2, 1965, pp. 923–927.

Chandrasekhar, Vol. 2, p. 925)

And again Chandrasekhar: *"Fermi's interest in hydromagnetic turbulence led him to inquire into the physics of ordinary hydrodynamic turbulence. Confessing ignorance of this subject, Fermi asked me (early in 1950) to come to his office and tell him about the ideas of Kolmogorov and Heisenberg which were then very much in the vogue. However, when I went to tell him, I found that it was not necessary for me to say beyond a few words: such as isotropy, the cascade of energy from large to small eddies etc. With only such words as clues, Fermi promptly went to the blackboard ("to see if I understand these words") and proceeded to derive the Kolmogorov spectrum for isotropic turbulence (in the inertial range) and the basis of Heisenberg's elementary theory. Fermi's manner of arguing is worth recording for its transparent simplicity.*

Divide the scale of $\log k$ *(where* k *denotes the wave number) into equal divisions, say* $(\ldots, n, n+1, \ldots)$. *In a stationary state the rate of flow of energy across "n" must be equal to the rate of flow across "n + 1." Therefore:*

$$E_{n,n+1} = \rho \frac{v_n}{k_n}(v_n k_n)^2 - \rho \frac{v_{n+1}}{k_{n+1}}(v_{n+1} k_{n+1})^2 \,, \tag{1}$$

if one remembers that the characteristic time associated with "eddies" with wave numbers in the interval $(n, n+1)$ *is* $(v_{n+1}k_{n+1})^{-1}$. *From this relation it follows that:*

$$v_n = Constant \times k_n^{-1/3} \,, \tag{2}$$

and this is equivalent to Kolmogorov's law. For decaying turbulence, equation (1) should be replaced by:

$$\frac{d}{dt}(\rho v_n)^2 = E_{n\,,n+1} \tag{3}$$

and this equation expresses the content of Heisenberg's theory." (Chandrasekhar, Vol. 2, pp. 925–926)

The paper FA 7 *E. Fermi, J. Pasta, S. Ulam: Studies of nonlinear problems* (266) (always quoted as FPU) is outstanding for several reasons: (a) It represents the first computer study of a nonlinear system; (b) the results contradicted the belief held since Poincaré, that any perturbed Hamiltonian system has to be chaotic. Fermi had considered it 'a little discovery' (as quoted by Ulam), thus immediately evaluating its extraordinary importance; (c) it was one of Fermi's last works, completed after his death in 1954; (d) remained unpublished for a decade; (e) coincides in time with Kolmogorov's theorem (1954), though FPU and Kolmogorov-Arnold-Moser (KAM) theory were linked to each other only in 1966; (f) inspired the discovery of solitons and numerous other studies; (g) its results are not fully understood till now and the FPU model continues its inspiring mission today, after half a century. In his recollections Ulam refers to Fermi's opinion on the importance of the "understanding of nonlinear systems" for the future fundamental theories, and the "potentialities of the electronic computing machines" and even mentions Fermi's learning of the actual coding (programming) during one summer. The FPU paper and its influence on various areas of astrophysics and stochastic dynamics are discussed in Appendix B (see the papers by A. Carati et al. (B.4), S. Ruffo (B.7) and G.M. Zaslavsky (B.11)). Here is the presentation written by S. Ulam.

"After the war, during one of his frequent summer visits to Los Alamos, Fermi became interested in the development and potentialities of the electronic computing machines. He held many discussions with me on the kind of future problems which could be studied through the use of such machines. We decided to try a selection of problems for heuristic work where in absence of closed analytic solutions experimental work on a computing machine would perhaps contribute to the understanding of properties of solutions. This could be particularly fruitful for problems involving the asymptotic-long time or "in the large" behavior of nonlinear physical systems. In addition, such experiments on computing machines would have at least the virtue of having the postulates clearly stated. This is not always the case in an actual physical object or model where all the assumptions are not perhaps explicitly recognized.

Fermi often expressed a belief that future fundamental theories in physics may involve nonlinear operators and equations, and that it would be useful to attempt practice in the mathematics needed for the understanding of nonlinear systems. The plan was then to start with the possibly simplest such physical model and to study the results of the calculation of its long-time behavior. Then one would gradually increase the generality and the complexity of the problem calculated on the machine. The Los Alamos report LA–1940 (paper No. 266) presents the results of the very first such attempt. We had planned the work in the summer of 1952 and performed the calculations the following summer. In the discussions preceding the setting up and running of the problem on the machine we had envisaged as the next problem a two-dimensional version of the first one. Then perhaps problems of pure kinematics, e.g., the motion of a chain of points subject only to constraints but no external forces, moving on a smooth plane convoluting and knotting itself indefinitely. These were to be studied preliminary to setting up ultimate models for motions of system where "mixing" and "turbulence" would be observed. The motivation was then to observe the rates of mixing and "thermalization" with the hope that the calculational results would provide hints for a future theory. One could venture a guess that one motive in the selection of problems could be traced to Fermi's early interest in the ergodic theory. In fact, his early paper (No. 11a) presents an important contribution to this theory.

It should be stated here that during one summer Fermi learned very rapidly how to program problems for the electronic computers and he not only could plan the general outline and construct the so-called flow diagram but would work out himself the actual coding of the whole problem in detail. The results of the calculations (performed on the old MANIAC machine) were interesting and quite surprising to Fermi. He expressed to me the opinion that they really constituted a little discovery in providing intimations that the prevalent beliefs in the universality of "mixing and *thermalization" in nonlinear systems may not be always justified.*

A few words about the subsequent history of this nonlinear problem. A number of other examples of such physical systems were examined by calculations on the electronic computing machines in 1956 and 1957. I presented the results of the original paper on several occasions at scientific meetings; they seemed to have aroused considerable interest among mathematicians and physicists and there is by now a small literature dealing with this problem. The most recent results are due to N.J. Zabusky. (i) His analytical work shows, by the way, a good agreement of the numerical computations with the continuous solution up to a point where a discontinuity developed in the derivatives and the analytical work had to be modified. One obtains from it another indication that the phenomenon discovered is not due to numerical accidents of the algorithm of the computing machine, but seems to constitute a real property of the dynamical system.

In 1961, on more modern and faster machines, the original problem was considered for still longer periods of time. It was found by J. Tuck and M. Menzel that after one continues the calculations from the first "return" of the system to its original condition the return is not complete. The total energy is concentrated again essentially in the first Fourier mode, but the remaining one or two percent of the total energy is in higher modes. If one continues the calculation, at the end of the next great cycle the error (deviation from the original initial condition) is greater and amounts to perhaps three percent.[13] *Continuing again one finds the deviation increasing—after eight great cycles the deviation amounts to some eight percent; but from that time on an opposite development takes place! After eight more, i.e., sixteen great cycles altogether, the system gets very close better than within one percent to the original state! This supercycle constitutes another surprising property of our nonlinear system."* (S.M. Ulam, Vol. 2, pp. 977–978)

Paper FA 8 *E. Fermi: Theories on the origin of the elements (240)* was a rough calculation of Fermi on the formation of the elements in the early hot big bang universe in response to Gamov's earlier attempt at solving this problem. It is followed by the later publication of the more detailed Fermi-Turkevich work on this problem, namely paper FA 9 *Fermi-Turkevich: An excerpt from "Theory of the origin and relative abundance distribution of the elements," by Ralph A. Alpher and Robert C. Herman.* These are discussed in detail in the companion book *Einstein, Fermi, Heisenberg and the Birth of Relativistic Astrophysics.*

[13](i) Exact Solutions for the Vibrations of a nonlinear continuous string. A. E. C. Research and Development Report, MATT-102, Plasma Physics Laboratory, Princeton University, October 1961.

1.3 Appendices

Appendix A includes some commentary articles on Fermi's resolution of this "4/3 problem" in the ratio between inertial mass and energy for the classical electron Coulomb field and a shorter journal article summarizing the natural completion of Fermi's original ideas about electromagnetic mass (see A.1–3), followed by a historical context commentary paper. We also reproduce the related article from 1949 by B. Kwal (see A.4) which seems to be the only one to touch upon this topic until the independent work of Rohrlich in 1960, after which Fermi's original contribution was rediscovered.

Appendix B contains a selection of the articles from the proceedings of the meeting "Fermi and Astrophysics" organized at the University of Rome "La Sapienza" and at the ICRANet Center in Pescara October 3–6, 2001 and published in *Il Nuovo Cimento B* **117**, Nos. 9–11 (2002). The meeting was focused on the influence of Fermi on astrophysics and general relativity: his activities related to these topics were clustered at the beginning and end of his scientific career. These articles, selected because of their direct commentary on articles by Fermi or related applications of his ideas expressed in those articles, are presented in alphabetical order of their first authors.

Susan Ames discusses the historical background of Fermi's work on cosmic rays, along with current problems and further prospects for the physics of cosmic rays. In particular she points out how the frequently discussed ultra-high cosmic rays cannot be accelerated by the Fermi mechanism. Equipartition between the energy of matter and that of cosmic rays was among the initial points made by Fermi, and in that context Ames mentions also the role of the cosmic microwave background radiation.

Donato Bini and Robert Jantzen give a summary of Fermi's discussion of what we now call Fermi coordinates and Fermi transport with a historical update including Walker's contribution which led to the terminology of "Fermi-Walker transport." This article explicitly estimates the various relativistic contributions to the Fermi-Walker transport for vectors around circular orbits in black hole spacetimes and in their Minkowski limit.

Dino Boccaletti comments on the two papers which resulted from the collaboration of Fermi with Chandrasehkar (see papers 261, 262 of Chapter 3). The first paper is devoted to the study of light dispersion in the polarization plane and using the effect to derive the galactic magnetic field. The second paper contains the generalization of the virial theorem in the presence of a magnetic field. The commentary notes that Fermi was the first scientist to draw attention to the possible existence of a galactic magnetic field.

The review of Andrea Carati, Luigi Galgani, Antonio Ponno and Antonio Giorgilli is devoted to the equipartition problem in the Fermi-Pasta-Ulam paradox both in classical and quantum mechanics. Equipartition is discussed starting from Planck's work and Poincaré's theorem. Numerical results on the dependence of the existence of equipartition and the corresponding time scales on a certain critical energy are mentioned.

Piero Cipriani reviews the work of Fermi in the field of classical analytical mechanics. After a short historical introduction, he emphasizes some aspects of geometrical methods of the description of dynamics and the theory of stochastic differential equations. Interesting recollections on Fermi are quoted.

John G. Kirk reviews the Fermi acceleration mechanism in the context of galactic nuclei and gamma ray bursts, i.e., in processes involving relativistic motion. Diffusive and non-diffusive versions of Fermi's stochastic acceleration are considered, including those predicting a softer spectrum of accelerated particles. The appearance of anisotropy in the accelerated particles with increasing gamma factor is discussed for various astrophysical situations.

Stefano Ruffo reviews evidence for long relaxation time scales in Hamiltonian systems, and shows how complex and diverse is the dynamics of long-range systems. The 'quasi-states' of Fermi-Pasta-Ulam are discussed particularly in the context of two theoretical approaches developed by the author and collaborators, one based on the Vlasov-Poisson equation, and the other based on the averaging of fast oscillations.

Costantino Sigismondi and Francesca Maiolino review an early work by Fermi completed on June 20, 1922, the year of his habilitation thesis on statistics at the Scuola Normale Superiore of Pisa, with an application to the case of comets. Fermi studied this case with a coplanar orbit to the one of Jupiter, neglecting the influence of other planets. The probability of ejection of the comet from the solar system (a parabolic or hyperbolic orbit) after interaction with Jupiter is calculated, as well as the probability of an impact with Jupiter. They apply Fermi's results to the case of the Earth in order to recover the time rate of collision of comets with our planet, which reliably produced the extinction of the dinosaurs. In this context the properties of the Oort cloud are discussed as well.

Costantino Sigismondi and Angelo Mastroianni recall that approximately in the same period Fermi studied the formation of X-ray images and presented his first experimental work as a dissertation at the University of Pisa in the spring of 1922. The need for Fermi to make an experimental essay was made mandatory since at that time theoretical physics was not yet considered sufficient to have independent validity. Although his seminal ideas are not among the bibliographical sources investigated by Riccardo Giacconi and Bruno Rossi (1960) when they proposed a telescope using X-rays, Fermi's thesis was the most complete study of X-ray physics in his time. Fermi used the technique of 'mandrels' to form optical surfaces. He anticipated the technique used for the mirrors of the Exosat, Beppo-SAX, Jet-X and XMM-Newton telescopes, a technique which is now a mainstay of optical manufacturing.

Alexei Yu. Smirnov reviews the neutrino flavor transformations in matter, as one of the authors of the original theoretical predictions and related observable effects. In particular, the Sudbury Neutrino Observatory results provide strong evidence of the neutrino flavor conversion. Neutrino conversion is discussed also in the context of supernova neutrinos and the corresponding predictions for the fluxes and energies at the Earth, including the role of the Earth matter effect. The author shows that the data of SN1987 can also be explained by the neutrino oscillations in the matter of Earth as conversions of muon and tau antineutrinos.

George M. Zaslavsky reviews the Fermi-Pasta-Ulam problem with an attempt to find the transition from regular to chaotic dynamics. The Fermi acceleration mechanism is considered as a precursor of the Fermi-Pasta-Ulam problem. The Kepler map introduced by Roald Sagdeev and George Zaslavsky and several other problems are considered, demonstrating the role of the Fermi-Pasta-Ulam work in the discretization methods of differential equations and in the study of chaotic systems when the Lyapunov exponent method is not efficient.