MAT2500-01/02 24S Quiz 9 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

The region R is bounded by the 3 curves $y=\frac{1}{2}x, x=0, x^2+y^2=4, y\geq 0$. Evaluate $\iint_R x\ y\ dA$. [Remember spaces for multiplication.]

a) in Cartesian coordinates as a single iterated double integral,

b) in polar coordinates (use an inverse trig function for the value of θ along the line $y=\frac{1}{2}x$),

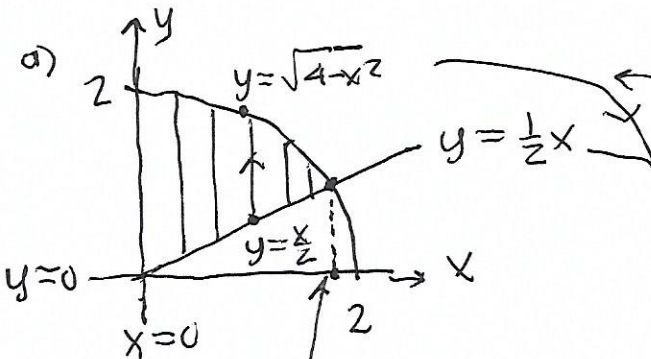
in each case accompanying your work by a new iteration diagram shaded by equally spaced linear cross-sections and a typical one with bullet point endpoints labeled by the equation of the starting and stopping values of the integration variable for the inner integral and with an arrowhead midway indicating the variable's increasing direction.

c) Use Maple to evaluate each such integral exactly. Do they agree as they should?

d) What is the average value of the integrand $f(x,y)=x\ y$ on this region numerically evaluated to 4 decimal places?

▶ **solution**

a)



$x^2+y^2=4 \rightarrow y=\pm\sqrt{4-x^2} \xrightarrow[y\geq 0]{} \sqrt{4-x^2}$

intersection: $x^2+\left(\frac{x}{2}\right)^2=4$

$x^2\underbrace{\left(1+\frac{1}{4}\right)}_{5/4}=4$

$x^2=\frac{16}{5} \rightarrow x=\frac{4}{\sqrt{5}}$

$y=\frac{x}{2}\ ..\ \sqrt{4-x^2}$ while $x=0\ ..\ \frac{4}{\sqrt{5}}$

$$\iint_R x y\ dA = \boxed{\int_0^{\frac{4}{\sqrt{5}}}\int_{\frac{x}{2}}^{\sqrt{4-x^2}} x\ y\ dy\ dx \overset{c)}{\underset{\text{Maple}}{=}} \frac{8}{5}}$$

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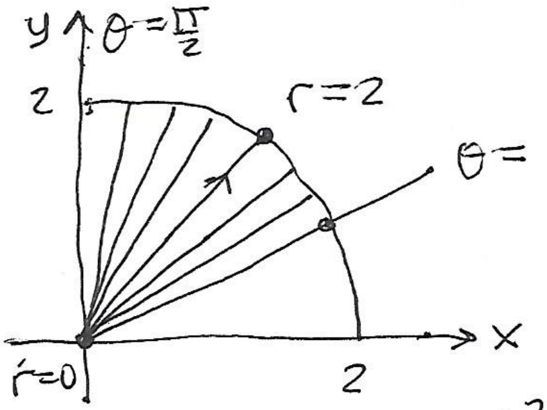
b) $x^2+y^2=4 \to r^2=4 \to r=2$

$x=0,\ y\ge 0 \to \theta=\pi/2$

$y=\frac{1}{2}x \to \frac{1}{2}=\frac{y}{x}=\tan\theta \to \theta=\arctan(1/2)$



$\Big\} \to r=0..2$ while $\theta=\arctan 1/2 .. \pi/2$

$$\iint_R xy\,dA=\int_{\arctan 1/2}^{\pi/2}\int_0^2 \underbrace{(r\cos\theta)}_{x}\underbrace{(r\sin\theta)}_{y}\underbrace{r\,dr\,d\theta}_{dA}$$

$$=\boxed{\int_{\arctan 1/2}^{\pi/2}\int_0^2 r^3\sin\theta\cos\theta\,dr\,d\theta} \underset{\text{Maple}}{\overset{\text{c)}}{=}} \boxed{\frac{8}{5}}$$

factors

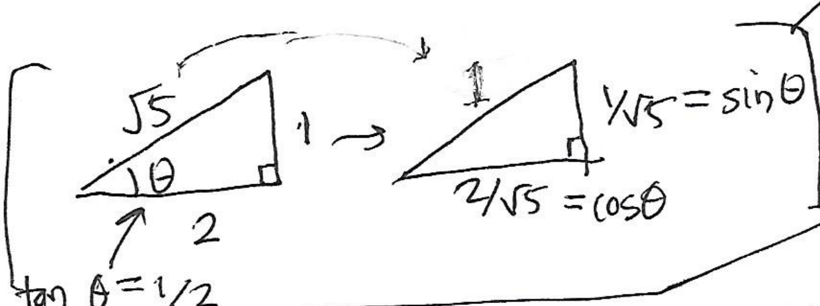
optional hand soln:

$$=\underbrace{\int_0^2 r^3\,dr}_{\frac{r^4}{4}\Big|_0^2=\frac{2^4}{4}=4}\ \underbrace{\int_{\arctan 1/2}^{\pi/2}\underbrace{\sin\theta}_{u}\ \underbrace{\cos\theta\,d\theta}_{du}}_{} \underset{\text{result}}{\overset{\text{final}}{=}} 4\left(\frac{2}{5}\right)=\frac{8}{5}$$

$\int u\,du=\frac{u^2}{2}+C$

$$\frac{1}{2}\sin^2\theta\Big|_{\arctan 1/2}^{\pi/2}=\frac{1}{2}\Big(\underbrace{\sin^2\tfrac{\pi}{2}}_{1}-\underbrace{\sin^2(\arctan 1/2)}_{(1/\sqrt5)^2}\Big)$$

$$=\frac{1}{2}\left(1-\frac{1}{5}\right)=\frac{1}{2}\left(\frac{4}{5}\right)=\frac{2}{5}$$



Area = (radius)($\Delta\theta$)

$=2(\pi/2-\arctan 1/2)$

d) $f_{avg}=\dfrac{8/5}{\text{Area}}=\dfrac{8/5}{2(\frac{\pi}{2}-\arctan 1/2)}=\boxed{\dfrac{4}{5(\pi/2-\arctan 1/2)}}$

or Area $=\int_{\arctan 1/2}^{\pi/2}\int_0^2 1\,r\,dr\,d\theta=\left(\int_0^2 r\,dr\right)\left(\int_{\arctan 1/2}^{\pi/2} 1\,d\theta\right)=\underbrace{\frac{r^2}{2}\Big|_0^2}_{2}\cdot\theta\Big|_{\arctan 1/2}^{\pi/2}$