MAT2500-05 24S Quiz 8 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. Find the volume under the surface $z = y^2 e^{x\,y}$ above the region bounded by $y = x$, $y = 4$ and $x = 0$. First set up the two iterated integrals representing this volume and in each case let Maple evaluate the double integral. Follow these steps:

a) Integrate first in the horizontal direction.

b) Integrate first in the vertical direction.

in the direction of the inner integration variable

In each case accompany your work with a new iteration diagram to justify your interation, a diagram shaded by equally spaced linear cross-sections and a typical one with bullet point endpoints labeled by the equation of the starting and stopping values of the integration variable for the inner integral and with an arrowhead midway indicating the variable's increasing direction. Consult the previous 22F quiz 8 answer key for a model of how to do this.

c) Use Maple to evaluate each such integral exactly. Do they agree as they should? [If you do this by hand, one of these avoids integration by parts.]

2. a) "Deconstruct" the integral $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1}\, dy\, dx$ by creating a diagram as above that explains the limits of integration in the current order in which the successive integrations are done.

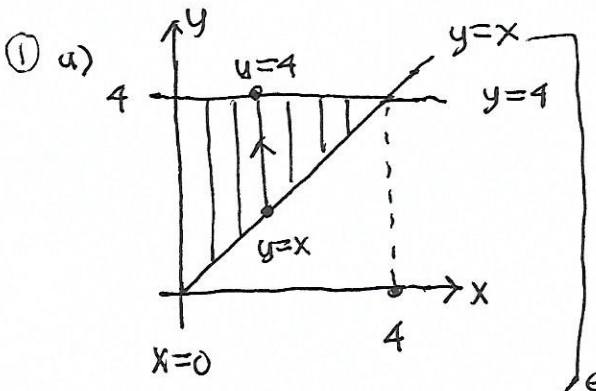
b) Create a new diagram explaining the limits for the reversed order of integration.

c) Using that diagram, write down the new iteration of this integral in that reversed order and evaluate it step by step by hand.

d) Check with Maple that the two iterations have the same numerical value. [Maple gets the first exact integration wrong, but correct numerical value of 0.4063171388 (I had to convert it to inert form to do this!), the second order only requires a u-substitution to finish the integration].

▶ solution

① a)



$y = x..4$ while $x = 0..4$

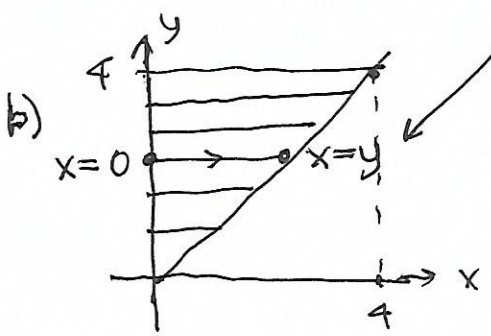
$$\int_0^4 \int_x^4 y^2 e^{xy}\, dy\, dx = \frac{1}{2}(e^{16} - 17)$$

Maple

$$\approx 4.144304670 \cdot 10^6$$

easy inversion!

b)



$x = 0..y$ while $y = 0..4$

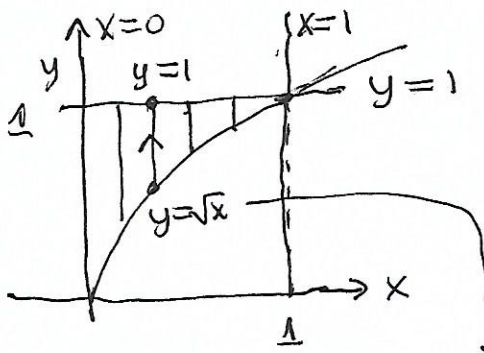
$$\int_0^4 \int_0^y y^2 e^{xy}\, dx\, dy = \text{Maple}$$

same as above

(this is the easy direction even for Maple)

MAT 2500-01/02 24S Quiz 8

② a) $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\, dy\, dx = \int_{x=0}^{x=1} \int_{y=\sqrt{x}}^{y=1} \sqrt{y^3+1}\, dy\, dx$



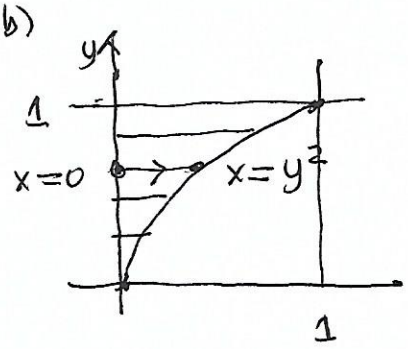
$y = \sqrt{x} .. 1$ while $x = 0..1$

start on parabola and move up.

invert

$y = x^{1/2} \rightarrow y^2 = x$

b)



$x = 0 .. y^2$ while $y = 0..1$

$$\int_0^1 \int_0^{y^2} \sqrt{y^3+1}\, dx\, dy$$

$$x\sqrt{y^3+1}\,\Big|_{x=0}^{x=y^2} = y^2(1+y^3)^{1/2} - 0$$

$$= \int_0^1 \underbrace{(1+y^3)}_{u}{}^{1/2} \underbrace{y^2 dy}_{\frac{du}{3}} = \int_{y=0}^{y=1} u^{1/2} \frac{du}{3}$$

$u = 1+y^3$

$du = 3y^2 dy$

$\frac{du}{3} = y^2 dy$

$$= \frac{u^{3/2}}{3/2} \cdot \frac{1}{3}\Big|_{y=0}^{y=1}$$

$$= \frac{2}{9}(1+y^3)^{3/2}\Big|_0^1$$

$$= \frac{2}{9}(1+1)^{3/2} - \frac{2}{9}(1) = \boxed{\frac{2}{9}(2^{3/2}-1)}$$

≈ 0.406317386 (8)

(Maple output: $-\frac{2}{9} + \frac{4\sqrt{2}}{9}$) when Maple does double integral directly