MAT2500-01/02 24s Quiz 7 Print Name (Last, First) _____|__
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. Consider $f(x,y) = x\,y - x^2\,y - x\,y^2$.
a) Set up the equations to determine its critical points.
b) Use Maple to find quickly the solutions.
c) Now derive these solutions by hand. [Hint: Factor $f_y(x,y)$ to get two conditions, use each in the other equation to finish the job.]
d) Next evaluate the second derivatives in general, and at the critical points you find. Make a table of values and annotate the entries like in this summary page:
https://www34.homepage.villanova.edu/robert.jantzen/courses/mat2500/handouts/2d2nddertest.pdf
linked to the HW page, and classify these points as local maxima, minima or saddle points, justifying your conclusions as indicated in this table.
e) Confirm that your classification of the 4 critical points looks right by using

```
> with(plots):
> contourplot(f(x,y), x=-1..2, y=-1..2, contours=500, gridlines=true)
> plot3d(f(x,y), x=-1..2, y=-1..2, view=-1..1, style=surfacecontour, contours=50)
```

No need to print out the plots (unless you wish to), just make sure your results are in agreement with what you see. Just say that they confirm your results (or do not if they do not).

2. Let $f(x,y) = x^2\,y^2 - y^3$, $P(1,2)$, $Q(-3,5)$.
a) Find the gradient of f and evaluate it at the point P.
b) Evaluate its magnitude and unit vector direction at this point.
c) Evaluate the directional derivative of f in the direction of Q at this point.
d) Derive the equation of the tangent plane at this point and simplify it to standard form.
e) Derive the vector equation of the normal line to the level surface through this point.

▶ solution