MAT2500-01/02 24S Quiz 7b

① $f(x,y) = xy - x^2y - xy^2$ $f(1,2) = 1\cdot 2 - 1^2\cdot 2 - 1\cdot 2^2 = 2-2-4 = -4$

a) $\frac{\partial f}{\partial x} = y - 2xy - y^2$

$\frac{\partial f}{\partial y} = x - x^2 - 2xy$

$\vec{\nabla} f(x,y) = \boxed{\langle y-2xy-y^2,\ x-x^2-2xy\rangle}$

$\vec{\nabla} f(1,2) = \langle 2 - 2\cdot 1\cdot 2 - 2^2,\ 1 - 1^2 - 2\cdot 1\cdot 2\rangle$

$= \langle 2-4-4,\ -4\rangle = \boxed{\langle -6,-4\rangle}$

b) $= 2\langle -3,-2\rangle$

$|\vec{\nabla} f(1,2)| = 2\sqrt{3^2+2^2} = \boxed{2\sqrt{13}} =$ max rate of change

$\hat{\nabla} f(1,2) = \boxed{\frac{\langle -3,-2\rangle}{\sqrt{13}}}$ direction of max rate of change

c) $\vec{PQ} = \vec{OQ} - \vec{OP} = \langle -3,5\rangle - \langle 1,2\rangle$

$= \langle -4,3\rangle$

$\hat{u} = \widehat{PQ} = \frac{\langle -4,3\rangle}{5}$ 3-4-5 triangle!

$D_{\hat{u}} f(1,2) = \hat{u}\cdot\vec{\nabla} f(1,2) = \frac{\langle -4,3\rangle}{5}\cdot 2\langle -3,-2\rangle$

$= \frac{2}{5}(4\cdot 3 - 3\cdot 2) = \frac{2\cdot 6}{5} = \boxed{\frac{12}{5}}$

d) $df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

$= (y-2xy-y^2)dx + (x-x^2-2xy)\,dy$

$df(1,2) = -6\,dx - 4\,dy$

$\langle 1.02, 1.99\rangle - \langle 1,2\rangle = \langle 0.02,-0.01\rangle = \langle dx,dy\rangle$

$df(1,2)\big|_{\substack{dx=0.02\\ dy=-0.01}} = -6(0.02) - 4(-0.01)$

$= -0.12 + 0.04 = \boxed{-0.08}$

e) $L(x,y) = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$

$= \boxed{-4 - 6(x-1) - 4(y-2)}$

$L(1.02, 1.99) = -4 - 6(1.02-1) - 4(1.99-2)$

$= -4 - 6(0.02) + 4(0.01)$

$\approx -4 - 0.12 + 0.04 = \boxed{-4.08}$

f decreases from -4 to -4.08 by the change -0.08!

f) $z = L(x,y)$

$= -4 - 6(x-1) - 4(y-2)$

$= -4 \underbrace{-6x+6-4y+8}_{10}$

$\boxed{6x + 4y + z = 10}$

$\vec{n} = \langle 6,4,1\rangle$

g) $\vec{r}_0 = \langle 1,2,-4\rangle$

$\vec{r} = \vec{r}_0 + t\,\vec{n}$

$= \langle 1,2,-4\rangle + t\langle 6,4,1\rangle$

$= \langle 1+6t,\ 2+4t,\ -4+t\rangle$

$\boxed{\langle x,y,z\rangle = \langle 1+6t,\ 2+4t,\ -4+t\rangle}$

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(2) $F(x,y,z) = x\sqrt{y^2+z^2} = x(y^2+z^2)^{1/2}$ $\qquad F(5,3,4) = 5\cdot\underbrace{\sqrt{3^2+4^2}}_{5} = 25$

a) $\dfrac{\partial F}{\partial x} = \sqrt{y^2+z^2}$

$\dfrac{\partial F}{\partial y} = x\cdot\frac{1}{2}(y^2+z^2)^{-1/2}\cdot 2y = \dfrac{xy}{\sqrt{y^2+z^2}}$

$\dfrac{\partial F}{\partial z} = x\cdot\frac{1}{2}(y^2+z^2)^{-1/2}\cdot 2z = \dfrac{xz}{\sqrt{y^2+z^2}}$

$\vec{\nabla}F(x,y,z) = \boxed{\left\langle \sqrt{y^2+z^2}, \dfrac{xy}{\sqrt{y^2+z^2}}, \dfrac{xz}{\sqrt{y^2+z^2}} \right\rangle}$

$\vec{\nabla}F(5,3,4) = \left\langle 5, \dfrac{5\cdot 3}{5}, \dfrac{5\cdot 4}{5} \right\rangle =$

$= \boxed{\langle 5,3,4\rangle}$

b) $dF(x,y,z) = \dfrac{\partial F}{\partial x}(xyz)\,dx + \dfrac{\partial F}{\partial y}(xyz)\,dy + \dfrac{\partial F}{\partial z}(xyz)\,dz$

$= \boxed{\sqrt{y^2+z^2}\,dx + \dfrac{xy}{\sqrt{y^2+z^2}}\,dy + \dfrac{xz}{\sqrt{y^2+z^2}}\,dz}$

$dF(5,3,4) = \boxed{5\,dx + 3\,dy + 4\,dz}$

$\vec{PQ} = \vec{OQ} - \vec{OP} = \langle 4.98-5, 3.01-3, 3.98-4\rangle$

$= \langle -0.02, 0.01, -0.02\rangle = \langle dx, dy, dz\rangle$

$dF(5,3,4)\Big|_{\substack{dx=-0.02\\ dy=0.01\\ dz=-0.02}} = 5(-0.02)+3(0.01)+4(-0.02)$

$= -0.10+0.03-0.08 = \boxed{-0.15}$

c) $L(x,y,z) = f(5,3,4) + f_x(5,3,4)(x-5) + f_y(5,3,4)(y-3) + f_z(5,3,4)(z-4)$

$\boxed{= 25 + 5(x-5) + 3(y-3) + 4(z-4)}$

$L(4.98, 3.01, 3.98) = 25 + 5(\underbrace{4.98-5}_{-0.02}) + 3(\underbrace{3.01-3}_{0.01}) + 4(\underbrace{3.98-4}_{-0.02})$

$= 25 - 5(0.02) + 3(0.01) - 4(0.02) = 25 - 0.10 + 0.03 - 0.08 = 25 - 0.15$

$= \boxed{24.85}$

f decreases by 0.15 to its approximate new value

d) $\boxed{x\sqrt{y^2+z^2} = 25} \leftarrow F(x,y,z) = F(5,3,4) \qquad \vec{r_0} = \langle 5,3,4\rangle$

$\vec{n} = \vec{\nabla}F(5,3,4) = \langle 5,3,4\rangle \rightarrow 0 = \vec{n}\cdot(\vec{r}-\vec{r_0}) = \langle 5,3,4\rangle\cdot\langle x-5, y-3, z-4\rangle$

$= 5(x-5)+3(y-3)+4(z-4) = 5x+3y+\underbrace{4z-5\cdot 5-3\cdot 3-4\cdot 4}_{-25-9-16\ =-50}$

$\boxed{5x+3y+4z=50}$

e) $L:\ \vec{r} = \vec{r_0} + t\vec{n} = \langle 5,3,4\rangle + t\langle 5,3,4\rangle$

$\boxed{\langle x,y,z\rangle = \langle 5+5t, 3+3t, 4+4t\rangle}$