MAT2500-01/02 24s Quiz 7b Print Name (Last, First) _____|__
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them. Show differentiation steps.

1. Consider $f(x,y) = x\,y - x^2\,y - x\,y^2$.
a) Evaluate the gradient of f in general, then at the point $P(1,2)$.
b) Evaluate the maximum rate of change at P and the unit direction in which f increases the most rapidly.
c) Evaluate the directional derivative of f at P in the direction towards the point Q(-3,5) from P using the appropriate symbol for this derivative.
d) Evaluate the differential of f in general (using the appropriate symbol for it), then at the point P. Use it to evaluate the approximate **change** in f from its value at P to its value at the nearby point (1.02,1.99). (2 decimal place accuracy)
e) Evaluate the linear approximation $L(x,y)$ to f at the point P: $L(x,y)$ =... (a function of x and y!). Use it to evaluate the approximate value of f at the nearby point (1.02,1.99). (2 decimal place accuracy)
f) Evaluate the equation for the tangent plane to the graph of f at the point $(1, 2, f(1,2))$ and reduce it to the standard form $a\,x + b\,y + c\,z = d$.
g) Obtain the vector equation $\langle x, y, z\rangle = \langle \ldots\rangle$ of the normal line to the graph of f at the point $(1, 2, f(1,2))$.

2. Let $F(x,y,z) = x\sqrt{y^2+z^2}$, $P(5, 3, 4)$, $Q(4.98, 3.01, 3.98)$.
a) Find the gradient of F and evaluate it at the point P.
b) Evaluate the differential of F in general, then at the point P. Use it to evaluate the approximate change in f from its value at P to its value at the nearby point Q (2 decimal place accuracy)
c) Evaluate the linear approximation $L(x,y,z)$ to F at the point P: $L(x,y,z)$ =... . Use it to evaluate the approximate value of F at the nearby point Q to 2 decimal places. Why is this consistent with the result of part b)?
d) State the equation of the level surface of F which passes through the point P and evaluate the equation for the tangent plane to that level surface at P and reduce it to the standard form $a\,x + b\,y + c\,z = d$.
e) Obtain the vector equation $\langle x, y, z\rangle = \langle \ldots\rangle$ of the normal line to the level surface of F at the point P.

► solution