MAT2500-01/02 24S Quiz 6 Print Name (Last, First) ________________________________|__
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.

1. a) Find the linear approximation of the function $f(x, y) = 1 - x\,y\cos(\pi y)$ at $(1, 1)$ and use it to approximate $f(1.02, 0.97)$.
b) Convince yourself that this is correct by making two Maple plots of the function graph and its tangent plane there (the graph of the linear approximation $L(x, y)$ at that point), first with $h = 1$ and then with $h = 0.01$, using the Maple command line below, inserting the appropriate expressions for these two functions. Rotate each plot around to verify that it looks right: the tangent plane appears to be tangent at the central point. In the zoom view can you see the very small separation between the graph and the tangent plane at the edges, or does it look like two different planes (which means it is wrong!)? [No need to print out the plots.]

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> h := 1 : plot3d([f(x, y), L(x, y)], x=1 - h..1 + h, y=1 - h..1 + h)
> h := 0.01 : plot3d([f(x, y), L(x, y)], x=1 - h..1 + h, y=1 - h..1 + h)
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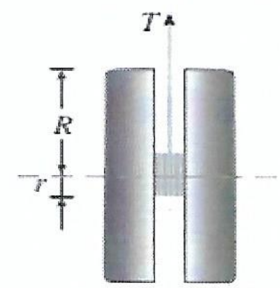
2. The tension in the attached string per unit mass of the yo-yo with inner and outer radii r and R respectively shown in the figure is given by $T = \dfrac{R}{2r^2 + R^2}$. Use differentials to estimate the change in the tension per unit mass if R is increased from 3 cm to 3.1 cm and is increased from 0.7 cm to 0.8 cm. Does the tension increase or decrease? State your differential expression dT at $(r, R) = (0.7, 3)$ to 3 significant figures and then its value for the appropriate differential values (dr, dR) to 3 significant figures in response to this question.



① a) $f(x,y) = 1 - xy\cos(\pi y)$ $\qquad f(1,1) = 1 - 1\cdot 1\cos\pi = 1 + 1 = 2$

$f_x = -y\cos\pi y$ $\qquad f_x(1,1) = -\cos\pi = 1$

$f_y = -x\cos\pi y + xy\sin(\pi y)\pi$ $\qquad f_y(1,1) = -\cos\pi + \pi\underbrace{\sin\pi}_{0} = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 2 + (x-1) + (y-1) = \boxed{x + y = L(x,y)}$

$L(1.02, 0.97) = 2 + (1.02 - 1) + (0.97 - 1)$

$= 2 + 0.02 - 0.03 = 2 - 0.01 = \boxed{1.99 \approx f(1.02, 0.97)}$

b) In the large view the tangent plane actually intersects the graph in a straight line, so it is not easy to see the point of tangency in the middle. In the zoomed view there is a very slight separation of the two surfaces on two corners.

MAT 2500-01/02 24S QUIZ 6 (2)

(2) $$T = \frac{R}{2r^2+R^2} = R(2r^2+R^2)^{-1}$$

$$\frac{\partial T}{\partial r} = R(-1)(2r^2+R^2)^{-2}(4r+0) = -\frac{4rR}{(2r^2+R^2)^2}$$

$$\frac{\partial T}{\partial R} = \frac{(2r^2+R^2)(1) - R(0+2R)}{(2r^2+R^2)^2} = \frac{2r^2+R^2-2R^2}{(2r^2+R^2)^2} = \frac{2r^2-R^2}{(2r^2+R^2)^2}$$

$$dT = \frac{\partial T}{\partial r}dr + \frac{\partial T}{\partial R}dR$$

$$= \frac{-4rR}{(2r^2+R^2)^2}dr + \frac{2r^2-R^2}{(2r^2+R^2)^2}dR = \frac{-4rR\,dr + (2r^2-R^2)\,dR}{(2r^2+R^2)^2}$$

$$dT\Big|_{\substack{r=0.7\\R=3}} = \frac{-4(0.7)(3)dr + (2\cdot 0.7^2 - 3^2)\,dR}{(2(0.7)^2+3^2)^2}$$

$$= \frac{-8.1\,dr - 8.02\,dR}{9.98} = \boxed{-(0.0843\,dr + 0.0805\,dR)}$$

$$dT\Big|_{\substack{r=0.7\\R=3\\dr=0.1\\dR=0.1}} = -(0.0843\cdot 0.1 + 0.0805\times 0.1) \approx \boxed{-0.0165}$$

$dr = 0.8-0.7 = 0.1$
$dR = 3.1-3 = 0.1$

$\boxed{\text{The tension (per unit weight) decreases.}}$

"mass" should have been "weight" mg in statement of problem. bob glitch!