MAT2500-01/02 24S Quiz 5

a) $\vec{r} = \langle \sin^3 t, \cos^3 t, \sin^2 t \rangle$

factor out

$\vec{r}\,' = \langle 3\sin^2 t\cos t, -3\cos^2 t\sin t, 2\sin t\cos t \rangle = \boxed{\sin t\cos t\,\langle 3\sin t, -3\cos t, 2\rangle} = \vec{V}$

$|\vec{r}\,'| = \sin t\cos t\sqrt{\underbrace{9\sin^2 t + 9\cos^2 t}_{9} + 4} = \boxed{\sqrt{13}\sin t\cos t} = v$

b) $s = \int_0^t \sqrt{13}\,\underbrace{\sin t}_{u}\,\underbrace{\cos t\,dt}_{du}$ or dummy $= \sqrt{13}\,\frac{\sin^2 t}{2}\Big|_0^t = \frac{\sqrt{13}}{2}(\sin^2 t - \sin^2 0) = \boxed{\frac{\sqrt{13}}{2}\sin^2 t}$

$s(\frac{\pi}{2}) = \frac{\sqrt{13}}{2}\sin^2(\frac{\pi}{2}) = \boxed{\frac{\sqrt{13}}{2}}$

c) $\hat{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\sin t\cos t\,\langle 3\sin t, -3\cos t, 2\rangle}{\sqrt{13}\sin t\cos t} = \boxed{\frac{1}{\sqrt{13}}\langle 3\sin t, -3\cos t, 2\rangle}$

d) $\hat{T}' = \frac{1}{\sqrt{13}}\langle 3\cos t, 3\sin t, 0\rangle = \underbrace{\frac{3}{\sqrt{13}}}_{|\hat{T}'|}\underbrace{\langle \cos t, \sin t, 0\rangle}_{\hat{N}}$ $\quad\boxed{\hat{N} = \langle \cos t, \sin t, 0\rangle}$

e) $\hat{B} = \hat{T}\times\hat{N} = \frac{1}{\sqrt{13}}\langle 3\sin t, -3\cos t, 2\rangle \times \langle \cos t, \sin t, 0\rangle$

$\underset{\text{maple}}{=} \boxed{\frac{1}{\sqrt{13}}\langle -2\sin t, 2\cos t, 3\rangle}$

e) $K = \frac{|\hat{T}'|}{|\vec{r}\,'|} = \frac{3/\sqrt{13}}{\sqrt{13}\sin t\cos t} = \boxed{\frac{3}{13}\csc t\sec t}$

$\rho = \frac{1}{K} = \boxed{\frac{13}{3}\sin t\cos t}$ ✓ ☹

f) $\vec{a} = \vec{r}\,'' = \boxed{\langle 6\sin t\cos^2 t - 3\sin^3 t, 6\cos t\sin^2 t - 3\cos^3 t, 2\cos^2 t - 2\sin^2 t\rangle}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$: $\vec{a}(\frac{\pi}{4}) = \langle (6-3)(\frac{1}{\sqrt{2}})^3, (6-3)(\frac{1}{\sqrt{2}})^3, 2(\frac{1}{2}) - 2(\frac{1}{2})\rangle$

$= \langle \frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}, 0\rangle = \boxed{\frac{3}{2\sqrt{2}}\langle 1, 1, 0\rangle}$

g) $a_{\hat{T}}(\frac{\pi}{4}) = \hat{T}(\frac{\pi}{4})\cdot\vec{a}(\frac{\pi}{4}) = \frac{1}{\sqrt{13}}\langle 3(\frac{1}{\sqrt{2}}), -3(\frac{1}{\sqrt{2}}), 2\rangle\cdot\langle 1, 1, 0\rangle\frac{3}{2\sqrt{2}}$

$= \frac{9}{2\sqrt{2}\sqrt{13}\sqrt{2}}(1 - 1 + 0) = \boxed{0}$

$a_{\hat{N}}(\frac{\pi}{4}) = \hat{N}(\frac{\pi}{4})\cdot\vec{a}(\frac{\pi}{4}) = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\rangle\cdot\langle 1, 1, 0\rangle\cdot\frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}\frac{1}{\sqrt{2}}(1+1) = \boxed{\frac{3}{2}}$

h) $|\vec{a}(\frac{\pi}{4})| = \left|\frac{3}{2\sqrt{2}}\langle 1, 1, 0\rangle\right| = \frac{3}{2\sqrt{2}}\cdot\sqrt{2} = \boxed{\frac{3}{2}}$ ✓

i) see Maple