Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.
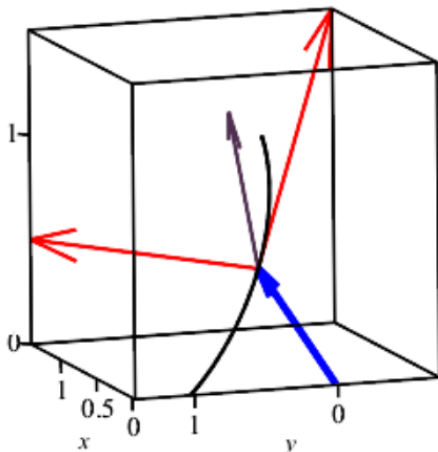
The parametrized curve segment $\vec{r}(t) = \langle \sin^3(t), \cos^3(t), \sin^2(t) \rangle$

, $0 \le t \le \dfrac{\pi}{2}$.

is shown in the figure together with $\vec{r}\left(\dfrac{\pi}{4}\right)$ and the first and second derivatives and binormal at the latter point on the curve. It is a perfect square arclength curve whose **T-N-B** frame components are all constants or constant multiples of $\sin(t)$ or $\cos(t)$, making it a good curve to use for hand evaluation of its geometry. **Factor out a common scalar factor of these three vectors to make them look simpler!**

a) Evaluate $\vec{v}(t) = \vec{r}'(t)$ and factor out a common scalar before evaluating and simplifying $v(t) = |\vec{r}'(t)|$,

b) This simplification allows you to find exactly evaluate the arclength function starting at $t=0$. Use it to evaluate the exact arclength of the curve over the interval $0 \le t \le \dfrac{\pi}{2}$.

c) Evaluate the unit tangent $\hat{T}(t)$.

d) Now evaluate $\hat{N}(t) = \dfrac{\hat{T}'(t)}{|\hat{T}'(t)|}$ (easy this time!) and then $B(t) = \hat{T}(t) \times \hat{N}(t)$ (simplify it!).

e) Evaluate the curvature $\kappa(t) = \dfrac{|\hat{T}'(t)|}{|\vec{r}'(t)|}$ and show that the radius of curvature is just $\rho(t) = \dfrac{13}{3} \sin(t)\cos(t)$

f) Evaluate $\vec{a}(t) = \vec{r}''(t)$ and its value at $t = \dfrac{\pi}{4}$.

g) Evaluate the scalar tangential projection $a_{\hat{T}}\left(\dfrac{\pi}{4}\right)$ along $\hat{T}\left(\dfrac{\pi}{4}\right)$ of the acceleration $\vec{a}\left(\dfrac{\pi}{4}\right) = \vec{r}''\left(\dfrac{\pi}{4}\right)$ and its scalar normal projection $a_{\hat{N}}\left(\dfrac{\pi}{4}\right) = \hat{N}\left(\dfrac{\pi}{4}\right) \cdot \vec{a}\left(\dfrac{\pi}{4}\right)$ exactly.

h) The magnitude of the acceleration is $a(t) = \sqrt{13 - 43\sin(t)^2\cos(t)^2}$. Does $a\left(\dfrac{\pi}{4}\right)^2 = a_{\hat{T}}\left(\dfrac{\pi}{4}\right)^2 + a_{\hat{N}}\left(\dfrac{\pi}{4}\right)^2$? Explain or demonstrate this.

i) **Optional.**
Use Maple to derive the magnitude of the acceleration $a(t)$ and show that it agrees with the above stated formula.

▶ **solution**