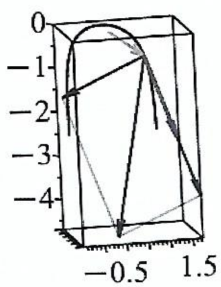
MAT2500-01/02 24S Quiz 4 Print Name (Last, First) ______________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.** Numerical values can be evaluated with technology.

Given the vector-valued function $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$ for the domain $-\frac{\pi}{2} < t < \frac{\pi}{2}$ where $\cos(t) > 0$:



a) Evaluate $\vec{r}\,'(t), \vec{r}\,''(t), |\vec{r}\,'(t)|, \hat{T}(t)$ and remember to simplify your results (no credit for unidentified expressions).

b) Evaluate $\vec{r}\left(\frac{\pi}{3}\right), \vec{r}\,'\left(\frac{\pi}{3}\right), \vec{r}\,''\left(\frac{\pi}{3}\right), \hat{T}\left(\frac{\pi}{3}\right)$ and remember to simplify your results (no credit for unidentified expressions).

c) Evaluate the exact angle θ in radians between $\vec{r}\,'\left(\frac{\pi}{3}\right)$ and $\vec{r}\,''\left(\frac{\pi}{3}\right)$ and a single decimal place approximation in degrees.

~~d) Evaluate the vector w which is the vector projection of $\vec{r}\,''\left(\frac{\pi}{3}\right)$ orthogonal to $\vec{r}\,'\left(\frac{\pi}{3}\right)$.,~~ Oops.

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e) Write the simplified equation for the plane through $\vec{r}\left(\frac{\pi}{3}\right)$ containing the first and second derivatives there as shown in the figure.

f) Evaluate the vector projections $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$ of $\vec{a} = \vec{r}\,''\left(\frac{\pi}{3}\right)$ along $\vec{r}\,'\left(\frac{\pi}{3}\right)$.

g) Evaluate their magnitudes $|\vec{a}|$, $|\vec{a}_{\parallel}|$ and $|\vec{a}_{\perp}|$. Do they satisfy the Pythagorean theorem?

a) $\vec{r} = \langle \cos t, \sin t, \ln(\cos t) \rangle$

$\vec{r}\,' = \langle -\sin t, \cos t, \frac{1}{\cos t}(-\sin t) \rangle$

$= \langle -\sin t, \cos t, -\tan t \rangle$

$|\vec{r}\,'| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t}$

$= \sqrt{1 + \frac{\sin^2 t}{\cos^2 t}} = \sqrt{\frac{\cos^2 t + \sin^2 t}{\cos^2 t}} = \frac{1}{\cos t}$

$= \sec t$

$\hat{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \cos t \langle -\sin t, \cos t, -\tan t \rangle$

$\vec{r}\,'' = \langle -\cos t, -\sin t, -\sec^2 t \rangle$

b) $\vec{r}\left(\frac{\pi}{3}\right) = \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, \ln\left(\frac{1}{2}\right) \rangle = \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, -\ln 2 \rangle$

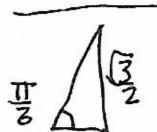
$\vec{r}\,'\left(\frac{\pi}{3}\right) = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3} \rangle$

$|\vec{r}\,'\left(\frac{\pi}{3}\right)| = 2$

$\hat{T}\left(\frac{\pi}{3}\right) = \frac{1}{2} \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3} \rangle$

$\vec{r}\,''\left(\frac{\pi}{3}\right) = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -4 \rangle$

b)



$\cos\frac{\pi}{3} = \frac{1}{2}, \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}, \sec\frac{\pi}{3} = 2$

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c) $\hat{T}(\frac{\pi}{3}) = \frac{1}{4}\langle -\sqrt{3}, 1, -2\sqrt{3}\rangle$

$|\vec{a}| = \left|\frac{1}{2}\langle -1, -\sqrt{3}, -8\rangle\right|$

$= \frac{1}{2}\sqrt{1+3+64} = \frac{1}{2}\sqrt{68} = \sqrt{17}$

$\hat{a} = \frac{1}{2\sqrt{17}}\langle -1, -\sqrt{3}, -8\rangle$

$\cos\theta = \hat{a}\cdot\hat{T}(\frac{\pi}{3}) = \frac{1}{4}\langle -\sqrt{3}, 1, -2\sqrt{3}\rangle \cdot \frac{1}{2\sqrt{17}}\langle -1, -\sqrt{3}, -8\rangle$

$= \frac{1}{8\sqrt{17}}(\cancel{\sqrt{3} - \sqrt{3}} + 16\sqrt{3}) = 2\sqrt{\frac{3}{17}}$

$\theta = \arccos\left(2\sqrt{\frac{3}{17}}\right) \approx \boxed{32.8^\circ}$

e) $\vec{r}\,'(\frac{\pi}{3})\times\vec{r}\,''(\frac{\pi}{3}) = \frac{1}{2}\langle -\sqrt{3}, 1, -2\sqrt{3}\rangle \times \frac{1}{2}\langle -1, -\sqrt{3}, -8\rangle$ $\quad \vec{r}_0 = \vec{r}(\frac{\pi}{3}) = \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, -\ln 2\rangle$

$= \frac{1}{4}\langle -14, -6\sqrt{3}, 4\rangle = -\frac{1}{2}\underbrace{\langle 7, 3\sqrt{3}, -2\rangle}_{\vec{n}}$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 7, 3\sqrt{3}, -2\rangle \cdot \langle x-\frac{1}{2}, y-\frac{\sqrt{3}}{2}, z+\ln 2\rangle$

$= 7(x-\frac{1}{2}) + 3\sqrt{3}(y-\frac{\sqrt{3}}{2}) - 2(z+\ln 2)$

$= 7x+3\sqrt{3}y-2z \underbrace{-\frac{7}{2}-\frac{9}{2}}_{=-8} - 2\ln 2 \to \boxed{7x+3\sqrt{3}y-2z = 8+2\ln 2}$

f) $\vec{a} = \frac{1}{2}\langle -1, -\sqrt{3}, -8\rangle$, $\hat{T}(\frac{\pi}{3}) = \frac{1}{4}\langle -\sqrt{3}, 1, -2\sqrt{3}\rangle$

$a_{\parallel} = \hat{T}(\frac{\pi}{3})\cdot\vec{a} = \frac{1}{8}(\sqrt{3} - \sqrt{3} + 16\sqrt{3}) = \boxed{2\sqrt{3} = |\vec{a}_{\parallel}|}$ g)

$\vec{a}_{\parallel} = a_{\parallel}\hat{T}(\frac{\pi}{3}) = 2\sqrt{3}\cdot\frac{1}{4}\langle -\sqrt{3}, 1, -2\sqrt{3}\rangle = \boxed{\langle -\frac{3}{2}, \frac{\sqrt{3}}{2}, -3\rangle}$

$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = \langle -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -4\rangle - \langle -\frac{3}{2}, \frac{\sqrt{3}}{2}, -3\rangle$

$= \langle -\frac{1}{2}+\frac{3}{2}, -\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}, -4+3\rangle = \boxed{\langle 1, -\sqrt{3}, -1\rangle}$

g) $|\vec{a}_{\perp}| = \sqrt{1+3+1} = \sqrt{5}$

$|\vec{a}_{\perp}|^2 + |\vec{a}_{\parallel}|^2 = (2\sqrt{3})^2 + (\sqrt{5})^2 = 4\cdot 3 + 5 = 17 = |\vec{a}|^2$ ✓