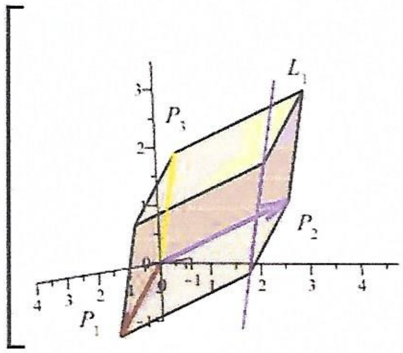
MAT2500-01/02 24S Quiz 3 Print Name (Last, First) ______________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.** Numerical values can be evaluated with technology.

Given three points $P_1(3,1,-1)$, $P_2(-1,2,1)$, $P_3(1,1,2)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$. [Note $\vec{r}_1$ comes forward to the front face of this object, with the parallelogram of $\vec{r}_2, \vec{r}_3$ equaling the back face.]



a) Write the parametrized equations of the (blue) line L_1 through the right edge of the front face $\mathscr{P}_{frontface}$ as shown. [What is the simplest position vector of a point on the line? What is the orientation of the line?] Where does this line intersect the $z=0$ plane?

b) Find a normal vector $\vec{n}$ for the plane $\mathscr{P}_{frontface}$ which contains the front face of the parallelepiped shown in the figure.

c) Write the simplified equation for this plane. Does the point at the tip of the main diagonal of the parallelopiped (from the origin to the opposite corner) satisfy this equation as it should?

d) Find the scalar projection h of the main diagonal of the parallelopiped along $\vec{n}$. [$|h|$ is just the distance of the front face plane from the origin, or its height if we instead think of that face as the top of the parallelopiped.]

e) Evaluate the area A of the front face of the parallelopiped, a parallelogram formed by the edges parallel to $\vec{r}_2, \vec{r}_3$.

f) Does the volume $V=|h|\,A$ of the parallelopiped equal the triple scalar product $|\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)|$ as it should?

a) $\vec{r}_0 = \vec{r}_1 + \vec{r}_2 = \langle 3,1,-1\rangle + \langle -1,2,1\rangle = \langle 2,3,0\rangle$

$\vec{a} = \vec{r}_3 = \langle 1,1,2\rangle$

$\boxed{\vec{r} = \vec{r}_0 + t\vec{a} = \langle 2,3,0\rangle + t\langle 1,1,2\rangle = \langle 2+t, 3+t, 2t\rangle = \langle x,y,z\rangle}$

or $\boxed{x=2+t,\ y=3+t,\ z=2t}$

$x=2,\ y=3 \leftarrow \overset{\|}{0} \to t=0$

L_1 intersects $z=0$ at point $\boxed{(2,3,0)}$

b) $\vec{r}_2 \times \vec{r}_3 = \langle -1,2,1\rangle \times \langle 1,1,2\rangle$

$= \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = \langle 4-1, 1+2, -1-2\rangle = \langle 3,3,-3\rangle$ (or use Maple) $= 3\langle 1,1,-1\rangle$

$\boxed{\vec{n} = \langle 1,1,-1\rangle}$

c) $\vec{r}_0 = \vec{r}_1 = \langle 3,1,-1\rangle$

$0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle 1,1,-1\rangle \cdot \langle x-3, y-1, z+1\rangle$

$= x-3+y-1-(z+1) = x+y-z-5$

$\boxed{x+y-z=5}$

part a)

$\vec{r}_{MD} = (\vec{r}_1+\vec{r}_2)+\vec{r}_3 = \langle 2,3,0\rangle + \langle 1,1,2\rangle = \langle 3,4,2\rangle$

$3+4-2=5$ ✓ yes, this point satisfies the equation

d) $|\vec{n}| = \sqrt{1+1+1} = \sqrt{3}$

$\hat{n} = \frac{1}{\sqrt{3}}\langle 1,1,-1\rangle$

$\vec{r}_{MD}\cdot\hat{n} = \langle 3,4,2\rangle \cdot \frac{\langle 1,1,-1\rangle}{\sqrt{3}}$

$= \frac{3+4-2}{\sqrt{3}} = \boxed{\frac{5}{\sqrt{3}} = h}$

e) $A = |\vec{r}_2 \times \vec{r}_3| = 3\,|\langle 1,1,-1\rangle| = \boxed{3\sqrt{3}}$

f) $V = |h|A = \frac{5}{\sqrt{3}}(3\sqrt{3}) = \boxed{15}$

$\vec{r}_1\cdot(\vec{r}_2\times\vec{r}_3) = \langle 3,1,-1\rangle\cdot\langle 3,3,-3\rangle$

$= 9+3+3 = 15$

$|\vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2)| = |\vec{r}_1\cdot(\vec{r}_2\times\vec{r}_3)| = 15$ ✓

yes, they agree.