

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). INDICATE where technology is used and what type (Maple, GC). Technology can only be used to check hand calculations and not substitute for them, unless specifically stated. Numerical values can be evaluated with technology.

1. Given the sphere described by the equation:

$$x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$$

- a) Complete the squares to find the radius a and the coordinates (x_0, y_0, z_0) of the center C of this sphere.
- b) Knowing the radius and center, how far is the sphere from the plane $z=4$. Explain. [Make a suggestive diagram if that helps.]
- c) What is the length of the position vector $|OC|$ of the center?
- d) Find a unit vector $u = \langle u_1, u_2, u_3 \rangle$ pointing from the origin to the center C .
- e) The horizontal plane through the center has the equation $z=z_0$. Substitute this into the equation of the sphere to determine the projection of its equator onto the x - y plane (a circle!).
- f) Use technology to implicit plot this circle, and make from it make a rough sketch of this circle in the x - y plane identifying its center with its coordinates (or print out a Maple ImplicitPlot) and annotate its center.

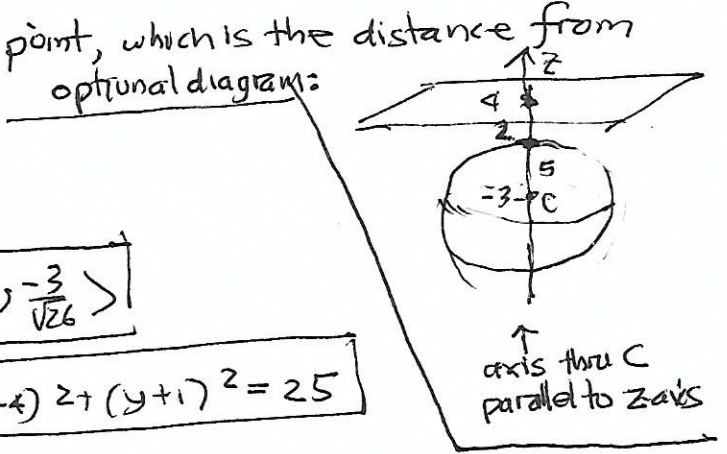
► solution

① a) $x^2 - 8x \rightarrow (x-4)^2 - 16$
 $+ y^2 + 2y \rightarrow (y+1)^2 - 1$
 $+ z^2 + 6z + 1 = 0 \rightarrow (z+3)^2 - 9 + 1 = 0$
 $-16 - 1 - 9 + 1 = -25$

$(x-4)^2 + (y+1)^2 + (z+3)^2 = 25 = 5^2$
 so $C(4, -1, -3)$ center and radius 5
 $\therefore x_0=4, y_0=-1, z_0=-3$

b) The North Pole of the sphere is the highest pt on it, a distance above the center equal to the radius at $z = -3 + 5 = 2$

The horizontal plane $z=4$ is 2 units above this point, which is the distance from the sphere to the plane: $d=2$

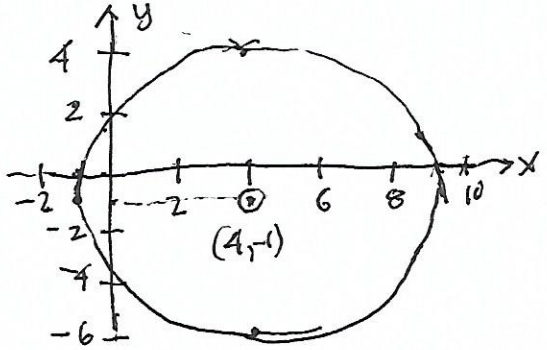


c) $\vec{OC} = \langle 4, -1, -3 \rangle$
 $|\vec{OC}| = \sqrt{4^2 + (-1)^2 + (-3)^2} = \sqrt{16+1+9} = \sqrt{26}$

d) $\hat{OC} = \frac{\vec{OC}}{|\vec{OC}|} = \frac{1}{\sqrt{26}} \langle 4, -1, -3 \rangle = \langle \frac{4}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{-3}{\sqrt{26}} \rangle$

e) substitute $z=-3$ or $z+3=0$ leads to $(x-4)^2 + (y+1)^2 = 25$

f) center at $(4, -1)$ radius 5:



a bit lame:
See Maple plot