MAT2500-01/02 24S Test 3 Print Name (Last, First) ___________________________________|__

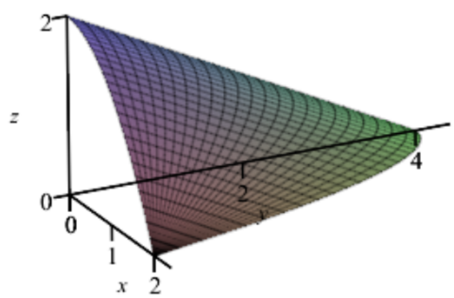
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

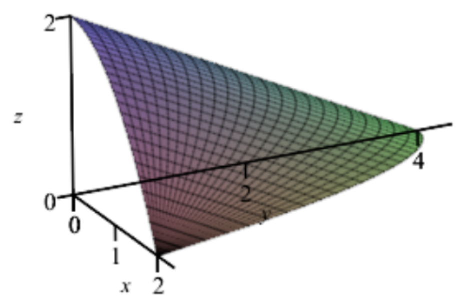
1. Consider the solid region R in the first octant between the coordinate planes (sides and bottom) $x=0, y=0, z=0$ and the surface (top) $x^2+y+2z=4$ pictured below. The integral over this region can be interated in 6 different orders. Using the upper plots below for each of the possible innermost integration variables, draw a typical linear cross-section in that direction with bullet endpoints labeled by the starting and stopping equations for that variable (arrow midway!) filling in the endpoint value below the graph, and for each responding outer double integral, make the usual diagram showing the two typical linear cross-sections representing the two possible orders of integration, as in the exercise we did in class for 15.6b.

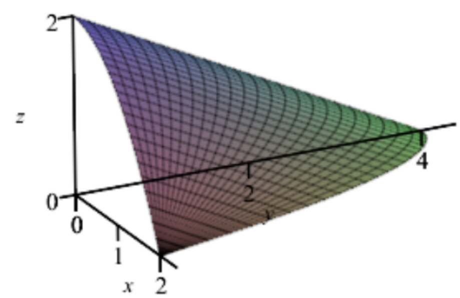
Use the diagrams below to justify the integral formulas for both choices, then evaluate the volume using Maple making sure all 6 values agree.

a) z first. b) $\boldsymbol{y}$ first. c) x first.

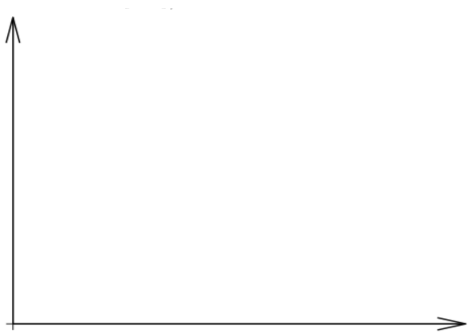
d) Choose one order to evaluate the coordinates of the centroid of this region exactly and give their decimal approximation to 2 decimal places. Does your result seem to be in the right position roughly?



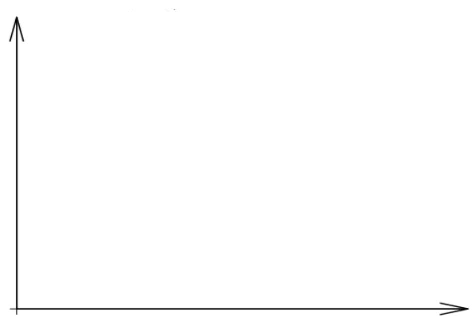




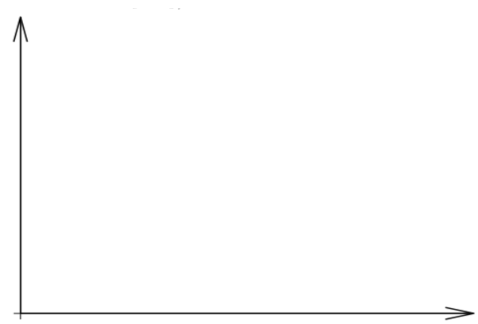
a) z first: $z=0$ ..   b) y first: $y=0$ ..   c) x first: $x=0$ ..



xy plane



xz plane



yz plane

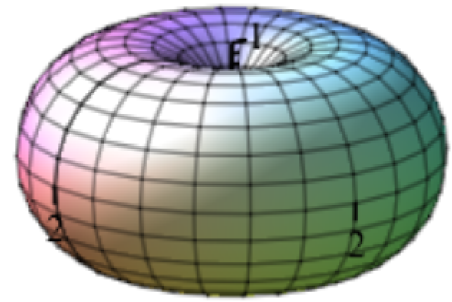
2. $\displaystyle\int_0^4\int_{\frac{y}{2}}^{\sqrt{y}} e^{3x^2-x^3}\,dx\,dy$ a) Evaluate this integral numerically to 6 digit accuracy using Maple.

b) Make a completely labeled diagram of the region of integration shaded by equally spaced linear cross-sections including one typical cross-section representing the current iteration of the integral (arrow midway) labeled by the starting and stopping values of the integration variable along that cross-section.

c) Make a new completely labeled diagram corresponding to the reversed order of integration.

d) State the new integral with the order of integration reversed.
e) Evaluate the new integral by hand step by step exactly (u-substitution) and give a numerical value for this exact result.
f) Do you get the same result as in part a)?

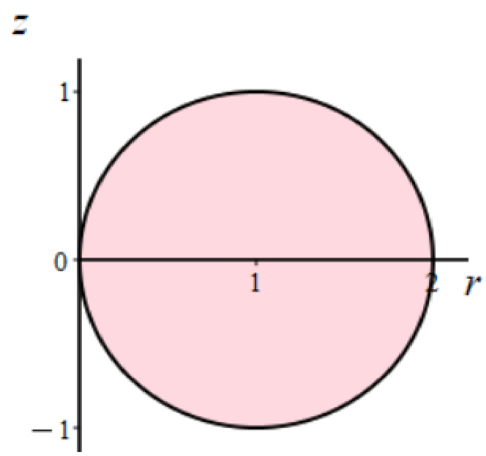
3. Consider the pinched torus described in cylindrical coordinates by $(r-1)^2 + z^2 = 1$ (equivalent to $r^2 + z^2 = 2r$), which encloses a solid region R of space shown in the figure, obtained by revolving a circle about the z-axis, as shown in the r-z diagrams below. Obviously $\theta = 0..2\pi$ for this solid of revolution and is the outer variable of integration of the triple integral. We analyze the inner double integral.



a) Express the triple integral $\iiint_R z^2 \, \mathrm{d}V$ in cylindrical coordinates, and evaluate using Maple.

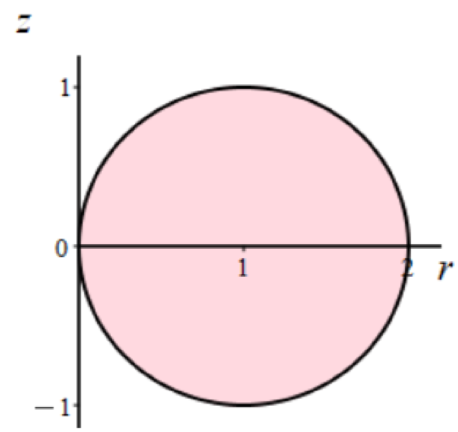
Justify your limits for the r-z inner double integral with the r-z half plane diagram below shaded by equally spaced lvertical linear cross-sections representing the inner integral, labeling one typical one as usual, and fill-in the starting and stopping values of the outer integration variable in that half plane.
b) Express the equation for this surface in spherical coordinates.
c) Express the triple integral $\iiint_R z^2 \, \mathrm{d}V$ in spherical coordinates, justified by a similarly labeled r-z half plane diagram below reresenting the limits of integration for ρ and ϕ, and evaluate using Maple. It should agree with a).
d) Evaluate the volume V of this region in spherical coordinates using Maple.
e) From c) and d) evaluate the average value of z^2 over this region.

cylindrical inner:



outer range: $r =$

spherical inner:



outer range: $\phi =$

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date: