MAT2500-01/02 24S Test 2 Print Name (Last, First) _____|_

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

▼ pledge

1. Consider the function $f(x, y) = x^3 - 6xy + 8y^3$. Identify all quantities you introduce by their proper symbols, as the instructions remind you.
a) What is the unit vector direction of maximum rate of change at the point (2,1)? The rate of change in that direction?
b) Evaluate the directional derivative in the direction of the nearby point (1.96,1.03).
c) Evaluate the linear approximation $L(x, y)$ at the point (2,1).
d) Use it to evaluate the approximate value at $(x, y) = (1.96, 1.03)$.
e) Simplify the equation of the tangent plane to the graph of f at (2,1) to the standard linear form $ax + by + cz = d$.
f) Write the simplified parametrized vector **equation** of the normal line to the graph of f at (2,1), using the symbols x,y,z.
g) Check that $(x, y) = (0, 0), \left(1, \dfrac{1}{2}\right)$ are critical points of this function and use the second derivative test to classify them as local maxima, minima or saddle points, supporting your claim with a few words or indications in addition to the derivative values that justify them. For local extrema, indicate what type of local extrema occur in the x and y directions before using the checking quantity to confirm or not a local extrema of the same type.

2. If the coordinates of a point are measured to be 3 m, 4 m, 12 m with a possible error in measurement of at most 0.2 cm = 0.002 m in each, use differentials to estimate the maximum error in the calculated value of the distance $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ from the origin in cm. Give your answer **in cm** to 2 decimal places. Show carefully all the steps in this process, identifying values of all the key quantities and using proper symbols for the quantities you introduce. State your response in a sentence giving the estimated maximum possible error in the calculated value of the distance with **cm** units.

► solution