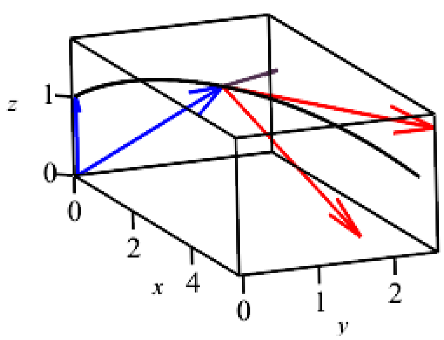
MAT2500-01/02 24S Test 1 Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:



The parametrized curve segment $\vec{\boldsymbol{r}}(t) = \left\langle \frac{t^2}{2}, \frac{1-t^3}{3}, \frac{t^3}{3} \right\rangle$, $0 \le t \le 2$ is shown in the figure together with $\vec{\boldsymbol{r}}(0)$ and $\vec{\boldsymbol{r}}(1)$ and the first and second derivatives at the latter point on the curve. It helps to keep overall denominators factored out of vector expressions.

a) Evaluate and simplify $\vec{\boldsymbol{v}}(t) = \vec{\boldsymbol{r}}\,'(t)$, $v(t) = |\vec{\boldsymbol{r}}\,'(t)|$.

b) Factoring out a common factor from inside the square root of part a) allows a u-substitution to evaluate its antiderivative. This allows you to exactly evaluate the arclength function starting at $t = 0$. Use it to evaluate the exact arclength of the curve over the interval $0 \le t \le 2$ and its numerical approximation to 4 decimal places.

c) Evaluate $\vec{\boldsymbol{a}}(t) = \vec{\boldsymbol{r}}\,''(t)$ and its magnitude $a(t)$ and their values at $t = 1$.

d) Evaluate the unit tangent $\hat{\boldsymbol{T}}(t)$ and $\hat{\boldsymbol{T}}(1)$.

e) Write the parametrized equations of the tangent line through $\vec{\boldsymbol{r}}(1)$.

f) Evaluate $\vec{B}(t) = \vec{\boldsymbol{r}}\,'(t) \times \vec{\boldsymbol{r}}\,''(t)$ and its magnitude and direction $\hat{\boldsymbol{B}}(t)$, the unit binormal, and its value at $t = 1$. Use it to write a simplified equation for the osculating plane at $t = 1$. Notice this is a plane curve (look at the second two components of the position vector). Is your result consistent with this? Explain.

g) Evaluate and simplify the unit normal $\hat{\boldsymbol{N}}(t) = \hat{\boldsymbol{B}}(t) \times \hat{\boldsymbol{T}}(t)$ and its value at $t = 1$.

h) Evaluate the curvature $\kappa(t) = \dfrac{|\vec{\boldsymbol{r}}\,'(t) \times \vec{\boldsymbol{r}}\,''(t)|}{|\vec{\boldsymbol{r}}\,'(t)|^3}$.

i) Evaluate the scalar tangential projection $a_{\hat{T}}(t)$ along $\hat{\boldsymbol{T}}(t)$ of the acceleration $\vec{\boldsymbol{a}}(t) = \vec{\boldsymbol{r}}\,''(t)$ and its scalar normal projection $a_{\hat{N}}(t) = \hat{\boldsymbol{N}}(t) \cdot \vec{\boldsymbol{a}}(t)$.

j) Does the normal acceleration satisfy $a_{\hat{N}}(t) = \kappa(t)\, v(t)^2$? Confirm.

To check dot products of vector functions and not be confused by the complex conjugate bug, load this package:
> with(VectorCalculus): BasisFormat(false):

▶ solution