MAT2705-04/05 23S Quiz 8 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.**

1. Consider the matrix $A = \begin{bmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{bmatrix}$.

a) Use Maple's Eigenvector command from the context menu to write down its real eigenvalues ordering them by increasing value and for each eigenvalue, write down Maple's basis of the eigenspace as a matrix B of columns in that same order.

b) Use Maple to evaluate the determinant $|A - \lambda\, Identity|$ to obtain the characteristic equation, then use Maple to solve it to get the eigenvalues.

c) For each eigenvalue, backsubsitute into the coefficient matrix $A - \lambda\, Identity$ and either reduce by hand or with Maple. Write out the reduced matrix equation and solve it using the algorithm. Compare your basis eigenvectors of each eigenspace with Maple's choice. Do they agree?

2. For the matrix $A = \begin{bmatrix} 6 & -17 \\ 8 & -6 \end{bmatrix}$, find the complex conjugate eigenvalues and corresponding complex conjugate eigenvectors by hand. Compare with Maple's result (recall the Maple uses $I = \sqrt{-1}$).

② $0 = |A - \lambda I| = \begin{vmatrix} 6-\lambda & -17 \\ 8 & -6-\lambda \end{vmatrix} = \underbrace{(6-\lambda)(-6-\lambda)}_{(\lambda+6)(\lambda-6)} + 8\cdot 17 = \lambda^2 - 36 + 136 = \lambda^2 + 100$

$\hookrightarrow \lambda = \pm 10i$

$\lambda = 10i$:

$$A - 10iI = \begin{bmatrix} 6-10i & -17 \\ 8 & -6-10i \end{bmatrix} \to \begin{bmatrix} 8 & -6-10i \\ 0 & 0 \end{bmatrix} \to \begin{array}{c} \begin{array}{cc} L & F \\ x_1 & x_2 \end{array} \\ \begin{bmatrix} 1 & -\frac{1}{4}(3+5i) \\ 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_2 = t$: $\quad x_1 - \frac{1}{4}(3+5i)x_2 = 0 \to x_1 = \frac{1}{4}(3+5i)t$

$0 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4}(3+5i)t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} \frac{1}{4}(3+5i) \\ 1 \end{bmatrix}}_{= b_1} \qquad b_2 = \overline{b_1} = \begin{bmatrix} \frac{1}{4}(3-5i) \\ 1 \end{bmatrix}$$

$$B = \langle b_1 | b_2 \rangle = \begin{bmatrix} \frac{1}{4}(3+5i) & \frac{1}{4}(3-5i) \\ 1 & 1 \end{bmatrix}$$

$\lambda = 10i, \ -10i$

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① a) Maple gives: $\lambda = -1, -1, 2$

$$B = \begin{array}{c} \begin{bmatrix} -1/2 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\ \;b_1 \quad b_2 \quad b_3 \end{array}$$

b) $$0 = |A - \lambda I| = \begin{vmatrix} 5-\lambda & -6 & 3 \\ 6 & -7-\lambda & 3 \\ 6 & -6 & 2-\lambda \end{vmatrix} \overset{\text{Maple}}{=} -\lambda^3 + 3\lambda + 2$$

$$\hookrightarrow \lambda = \underbrace{-1, -1}_{m=2}, 2$$

c) $\lambda = -1$:

$$A + I = \begin{bmatrix} 5+1 & -6 & 3 \\ 6 & -7+1 & 3 \\ 6 & -6 & 2+1 \end{bmatrix} = \begin{bmatrix} 6 & -6 & 3 \\ 6 & -6 & 3 \\ 6 & -6 & 3 \end{bmatrix} \to \begin{array}{c} \begin{bmatrix} 1 & -1 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ x_1 \; x_2 \; x_3 \\ L \;\; F \;\; F \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = t_1$ $\qquad x_1 - x_2 + \frac{1}{2} x_3 = 0 \;\to\; x_1 = t_1 - \frac{1}{2} t_2$

$x_3 = t_2$ $\qquad 0 = 0$

$\qquad\qquad 0 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t_1 - \frac{1}{2} t_2 \\ t_1 \\ t_2 \end{bmatrix} = t_1 \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}_{=b_1} + t_2 \underbrace{\begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}}_{=b_2}$$

$\lambda = 2$:

$$A - 2I = \begin{bmatrix} 5-2 & -6 & 3 \\ 6 & -7-2 & 3 \\ 6 & -6 & 2-2 \end{bmatrix} = \begin{bmatrix} 3 & -6 & 3 \\ 6 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 1 \\ 1 & -1 & 0 \end{bmatrix} \to \begin{bmatrix} 0 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \\ x_1 \; x_2 \; x_3 \\ L \;\; L \;\; F \end{array}$$

$x_1 - x_3 = 0$ $\qquad x_1 = t$

$x_2 - x_3 = 0$ $\qquad x_2 = t$

$0 = 0$

$\to x_3 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{b_3}$$

so my basis changing matrix is instead

$$B = \langle b_1 | b_2 | b_3 \rangle = \begin{array}{c} \lambda = -1, -1, 2 \\ \begin{bmatrix} 1 & -1/2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

interchanged wrt Maple (but random!)