MAT2705-04/05 23s Quiz 7 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.**

Consider the underdamped oscillator IVP: $4x''+4x'+17x=0, x(0)=4, x'(0)=-8$ [independent variable: t].

a) Put this into standard form by dividing through by the leading coefficient 4 and read off the exact and numerical values of k_0, τ_0, ω_0 and the numerical value of the quality factor $Q=\omega_0\tau_0$. (2 decimal places are sufficient.)

b) Find the general solution. What are the exact and approximate values of the characteristic equation root parameters k, ω and the two related time scales: $r=-k\pm i\omega, \tau=\frac{1}{k}, T=\frac{2\pi}{\omega}$.

c) Find the solution satisfying the initial conditions.

d) Re-express your solution with its sinusoidal factor in phase-shifted cosine form. Identify the initial amplitude A_0 and phase shift δ chosen in the interval $-\pi<\delta\le\pi$ and the numerical value of the fractional phase shift $\frac{\delta}{2\pi}$ (2 decimal places) to understand what fraction of the period the peak of the sinusoidal factor is shifted from $t=0$. Does the sinusoidal factor peak shift left or right from the vertical axis at $t=0$?

e) Use this plotting template below choosing the horizontal range appropriately ($t=0..5\tau$) to plot the solution of part c) (not d)!) with its envelope functions $x=\pm A_0 e^{-kt}$ and print it on paper and attach to this quiz. Make sure you put an asterisk before the parenthesis after the exponential.

$$> plot\left(\left[e^{-\frac{t}{5}}\cdot(12\cos(2t)+5\sin(2t)), 13e^{-\frac{t}{5}}, -13e^{-\frac{t}{5}}\right], t=0..25, color=black\right)$$

Optional. Plot the difference of c) and d) to make sure it is (numerically approximately) zero.

a) $4x''+4x'+17x=0$

$x''+x'+\frac{17}{4}x=0$

$\omega_0=\sqrt{\frac{17}{4}}=\frac{\sqrt{17}}{2}\approx 2.06$

$\boxed{k_0=1 \quad \tau_0=1/k_0=1 \quad Q=\omega_0\tau_0\approx 2.06>0.5}$ (underdamped)

b) $x=e^{rt}$: $4r^2+4r+17=0$

$r=\frac{-4\pm\sqrt{16-4\cdot4\cdot17}}{2\cdot4}=\frac{-1\pm\sqrt{-16}}{2}$

$=-\frac{1}{2}\pm 2i \rightarrow \boxed{k=\frac{1}{2}, \omega=2}$

$\boxed{T=\frac{2\pi}{2}=\pi\approx 3.14}$ $\boxed{\tau=\frac{1}{k}=2}$

b) continued. $e^{rt}=e^{(-\frac{1}{2}\pm 2i)t}=e^{-\frac{t}{2}}(\cos 2t\pm i\sin 2t)\rightarrow$ basis: $e^{-\frac{t}{2}}\cos 2t, e^{-\frac{t}{2}}\sin 2t$

gen soln: $\boxed{x=e^{-\frac{t}{2}}(c_1\cos 2t+c_2\sin 2t)}$

c) $x'=-\frac{1}{2}e^{-\frac{t}{2}}(c_1\cos 2t+c_2\sin 2t)+e^{-t/2}(-2c_1\sin 2t+2c_2\cos 2t)$

$x(0)=c_1=4$

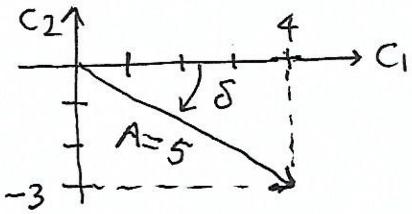
$x'(0)=-\frac{1}{2}c_1+2c_2=-8$

$c_2=\frac{1}{2}(-8+\frac{1}{2}(4))=-3$

$\boxed{x=e^{-t/2}(4\cos 2t-3\sin 2t)}$

MAT2705-04/05 23S Quiz 7

d) $(c_1, c_2) = (4, -3)$



$A = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$\tan\delta = -\frac{3}{4} \rightarrow \delta = -\arctan\frac{3}{4}$

$$\boxed{x = 5e^{-t/2}\cos(2t + \arctan 3/4)}$$

$\frac{\delta}{2\pi} = -\frac{1}{2\pi}\arctan 3/4 \approx -0.10$

The cosine peak shifts $\boxed{\text{left}}$ about 1/10 of a cycle (period).

e) decay window for exponential: $0 \le t \le 5\tau = 10$

plot: $\pm 5e^{-t/2}$, $e^{-t/2}(4\cos 2t - 3\sin 2t)$

> $plot\left(\left[e^{-\frac{t}{2}}(-3\sin(2t)+4\cos(2t)), 5e^{-\frac{t}{2}}, -5e^{-\frac{t}{2}}\right], t=0..10, color=black\right)$