

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.**

1. a) Although this DE is separable, use the linear solution approach to find its general solution

$$(x^2 + 4) y' = -3xy + x, \quad y(0) = 1$$

including every step by hand.

b) Then find the solution satisfying the initial condition. Does your solution agree with Maple, yes or no?

c) To check your solution, back substitute this initial value problem solution $y = y(x)$ and its derivative into the original DE and simplify both sides without changing their values until both sides are the same.

2. a) A 30 year old woman accepts an engineering position with a starting salary of \$60,000 per year. Her annual salary $S(t)$ increases exponentially with $S(t) = 60 e^{0.05t}$ thousand dollars after t years. Meanwhile, 10% of her salary is deposited continuously in a retirement account, which accumulates interest at a continuous annual rate of 8%. The amount $A(t)$ in her retirement account is described by the initial value problem:

$$\frac{dA}{dt} = 0.10 \cdot 60 e^{0.05t} + 0.08 A, \quad A(0) = 0.$$

Use the linear solution approach to find the general solution, and then the IVP solution, and finally the amount $A(30)$ when she early retires at age 60. Give your final answer to the nearest thousand dollars.

b) What is the characteristic time in years for the exponential growth of her income? What is the doubling time? [One decimal place is sufficient.]

► solution