Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.

1. $(4 - x^2) \dfrac{dy}{dx} = -4 x\, y$

initial condition 1: $y(-1) = \dfrac{1}{2}$;

initial condition 2: $y(-1) = 0$;

a) Indicate these initial data points on the graph by circled dots annotated by their (x, y) values and roughly draw in the corresponding solution curves which pass through them. Estimate your value for $y\left(\dfrac{3}{2}\right)$.

$y\left(-\tfrac{1}{2}\right) = \tfrac{1}{2}$

b) Find the general solution of the differential equation. Does your formula for the solution work for the second initial condition? If not, redefine your final constant.

c) Find the solution which satisfies the first initial condition.

d) Evaluate $y\left(\dfrac{3}{2}\right)$ for the first initial condition solution exactly and to 2 decimal places for this solution and mark the corresponding point on the graph by a circled dot.

$y(-1) = 0$

$y\left(\tfrac{3}{2}\right) \approx 0.17$

$y\left(\tfrac{3}{2}\right) \approx 0.20$ on curve
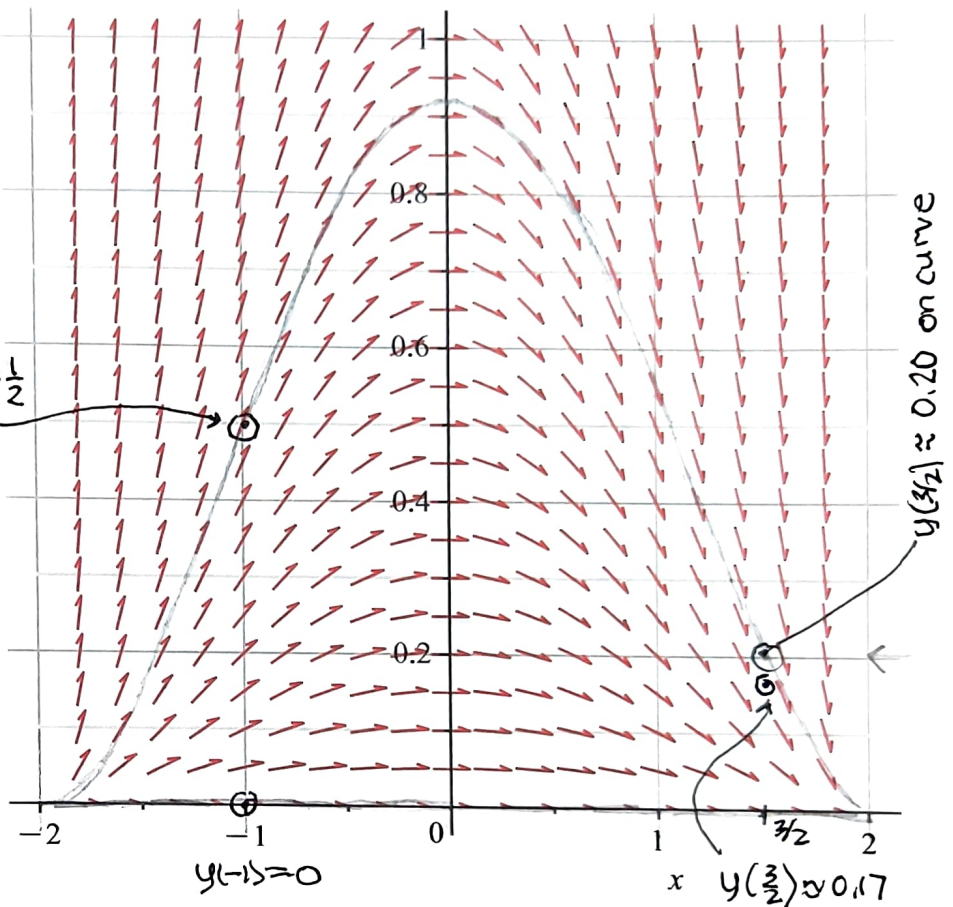
Is this consistent with your approximate hand drawn solution? Explain. Do not redraw your curve!

e) Check by hand that your solution to c) solves the differential equation by first calculating y' and subsituting simultaneously into the original DE for both y and y', then independently simplifying both sides.

f) Enter the differential equation and the initial condition $y(-1) = \dfrac{1}{2}$ separated by a comma in Maple. Use the context sensitive menu to solve. Write down the form of the solution that it gives you.

► **solution**

b) $(4-x^2)\dfrac{dy}{dx} = -4xy$

$\displaystyle\int\dfrac{dy}{y} = \int\dfrac{-4x\,dx}{4-x^2} = 2\int\dfrac{-2x\,dx}{4-x^2} = 2\int\dfrac{du}{u} = 2\ln|u| + C_1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 2\ln|4-x^2| + C_1$

$\ln|y| =$

$\qquad\qquad\qquad\qquad u = 4-x^2$
$\qquad\qquad\qquad\qquad du = -2x\,dx$

$|y| = e^{\ln|y|} = e^{2\ln|4-x^2| + C_1} = e^{C_1}\left(e^{\ln|4-x^2|}\right)^2$

$\qquad\qquad = e^{C_1}|4-x^2|^2 \swarrow$ even power

$y = \underbrace{\pm e^{C_1}}_{C}(4-x^2)^2$

$\boxed{y = C(4-x^2)^2}$ gen soln   (includes soln $y=0$ for $C=0$)

c) $y(-1) = \frac{1}{2} \Leftrightarrow x = -1, y = \frac{1}{2}$

$\frac{1}{2} = C(4-(-1)^2)^2 = C\cdot 3^2 = 9C \to C = \frac{1}{18}$

$\boxed{y = \frac{1}{18}(4-x^2)^2}$ IVP soln

d) $y(\frac{3}{2}) = \frac{1}{18}\left(4 - \underbrace{(\frac{3}{2})^2}_{\frac{9}{4}}\right)^2 = \frac{1}{18}\left(\dfrac{16-9}{4}\right)^2 = \dfrac{49}{18\cdot 16} = \boxed{\dfrac{49}{288}}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \approx 0.1701$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 2 dec places $\approx \boxed{0.17}$

$\qquad\qquad\qquad\qquad\qquad\qquad$ compare $\to$ 0.20 graphical estimate

$\qquad\qquad\qquad\qquad\qquad\qquad$ a bit off, hand drawn curve, not very precise!

e) $y = \frac{1}{18}(4-x^2)^2$

$\dfrac{dy}{dx} = \frac{1}{18}\cdot 2(4-x^2)(-2x) = -\frac{2x}{9}(4-x^2)$

substitute both into original DE then simplify both sides independently

$(4-x^2)\dfrac{dy}{dx} = -4x\left(\frac{1}{18}\cdot(4-x^2)^2\right)$

$(4-x^2)\left(-\frac{2x}{9}(4-x^2)\right) = -\frac{2x}{9}(4-x^2)^2$

$-\frac{2x}{9}(4-x^2)^2 = -\frac{2x}{9}(4-x^2)^2$ ✓

f) $y(x) = \dfrac{(x^2-4)^2}{18}$ is the Maple output ✓   note $(x^2-4)^2 = (4-x^2)^2$!