MAT2705-04/05 23S Quiz 1 Print Name (Last, First) ______________________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

1. $x \frac{dy}{dx} - 3\,y = x^3$, *soln* : $y = x^3\,(C + \ln(x))$

a) Verify that this y satisfies the given differential equation.

b) Find the solution which satisfies the initial condition $y(\mathrm{e}) = 2$. Simplify your result so no fractions remain in the expression for y. (Use rules of exponents and ln's. Maple's solution does this automatically!)

Organize your work as though you were playing professor.

2. a) Write a differential equation that models the situation:

"The line tangent to the graph of $g(x)$ at the point (x, y) intersects the x-axis at the point. $\left(\frac{x}{2}, 0\right)$."

Optional: Can you draw a "generic diagram" containing the two points in order to calculate the slope of the tangent line? (Check the answer key once public if not!)

b) Use Maple to solve this DE. Write down Maple's solution: $y = g(x)$.

▶ **solution**

① a) $y = x^3(C + \ln x)$

$$\frac{dy}{dx} = 3x^2(C+\ln x) + x^3\left(0 + \frac{1}{x}\right)$$
$$= 3x^2(C+\ln x) + x^2$$

$$x\frac{dy}{dx} - 3y = x^3$$
$$x\left(3x^2(C+\ln x) + x^2\right) - 3\,x^3(C+\ln x) = x^3$$
$$\underbrace{3x^3(C+\ln x) + x^3 - 3x^3(C+\ln x)}_{0} = x^3$$
$$x^3 = x^3 \checkmark$$

b) $2 = y(e) = e^3(C + \underbrace{\ln e}_{1})$

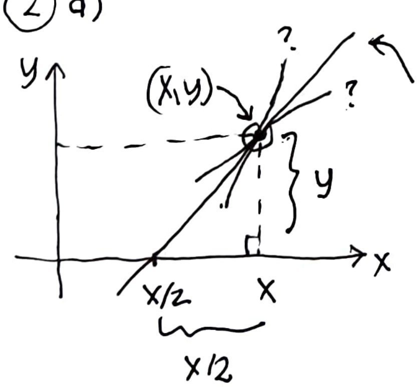
$2e^{-3} = C + 1$

$C = 2e^{-3} + 1$

$\boxed{y = x^3\left(2e^{-3} + 1 + \ln x\right)}$

Agrees with Maple!

② a)



$m = \dfrac{y}{x/2} = \dfrac{2y}{x} = \dfrac{dy}{dx}$ so $\boxed{\dfrac{dy}{dx} = \dfrac{2y}{x}}$

b) Maple gives : $\boxed{y = Cx^2}$

(literally $y(x) = -C_1 x^2$)

Firstevaluate $\frac{dy}{dx}$, then replace simultaneously y and $\frac{dy}{dx}$ in the original DE, then simplify both sides independently! Do not manipulate the DE before or after by mixing both sides in equation solving steps. This can introduce new errors.