

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses without showing details (identify technology).** Otherwise only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.

## pledge [sign and date the pledge at the end of your exam]

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature:

Date:

Consider the initial value problem: [START by finding Maple's solution.]

$$x_1'' = -7x_1 + 4x_2 + 5\cos(2t),$$

$$x_2'' = -3x_1 - 3x_2 + 5\cos(2t),$$

$$x_1(0) = -1, x_2(0) = 2,$$

$$x_1'(0) = -20, x_2'(0) = 6$$

Solve this problem following these steps. To make sure you copy down the numbers correctly in working this problem, know that the final expressions for the unknowns only involve single digit integers, although you encounter fractions along the way.

a) Write this system of DEs and initial conditions in matrix form, identifying the coefficient matrix  $A$ . and the driving force  $F$ .

**Bypass hand calcs here but not on final exam: USE MAPLE to get  $B$  etc:**

b) Find the eigenvalues and eigenvectors of the coefficient matrix by hand showing all steps in the matrix solution process (identifying leading and free variables, stating the reduced matrix equations, etc.) and order the eigenvalues by increasing absolute value  $|\lambda_1| < |\lambda_2|$  and chose the smallest integer component eigenvectors in the basis changing matrix  $B = \langle b_1 | b_2 \rangle$ . State its inverse  $B^{-1}$  and the diagonalized matrix  $A_B = B^{-1} A B$  and calculate by hand  $F_B = B^{-1} F$ .

c) Introduce the decoupling change of variables  $x = B y = y_1 b_1 + y_2 b_2$  and re-express the matrix DE in the new variables, and then write down the corresponding decoupled scalar DEs for the new variables  $y_1, y_2$ . **[Jump to  $y'' = A_b y + F_B$ ]**

d) Write down the general homogenous solutions of each decoupled DE.

e) Find the particular solution of each decoupled DE using the method of undetermined coefficients.

f) Write down the general solution for the original variables in matrix or vector form (as a linear combination) or in scalar form as you wish.

g) Impose the initial conditions on this general solution and express the resulting solution in scalar form for each of the unknowns.

h) Now rewrite your final solution in the following linear combination vector form by re-expressing the homogeneous solutions  $y_{1h}, y_{2h}$  in exact phase-shifted cosine form

$$x = A_1 \cos(\omega_1 t - \delta_1) b_1 + A_2 \cos(\omega_2 t - \delta_2) b_2 + \cos(2t) b_3,$$

supporting your two amplitude-phase-shift calculations by clearly labeled diagrams, where  $b_3$  is the vector coefficient of  $\cos(2t)$  that corresponds to the particular solution in response to the driving force.

i) On the grid provided plot the two eigenvectors and the new coordinate axes (label everything), and then the 6 vectors  $\pm A_1 b_1, \pm A_2 b_2, \pm b_3$

and finally the parallelogram whose sides correspond to  $y_1 = \pm A_1, y_2 = \pm A_2$ . [This parallelogram confines the homogeneous solution.]

