MAT2705-04/05 23S Test 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **You may use technology for row reductions, determinants and matrix inverses without showing details (identify technology).** Otherwise only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

pledge **[sign and date the pledge at the end of your exam]**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. a) Find the solution of the initial value problem $x'' + 4x' + 5x = 10\cos(3t), x(0) = 0, x'(0) = 0$ step by step, first finding the homogeneous solution, then the particular solution, then the general solution, and finally the solution satisfying the inital conditions. Justify all steps clearly.

b) Express the steady state solution in exact phase-shifted cosine form with the phase shift $-\pi \le \delta \le \pi$.

c) Express the transient solution in exact phase-shifted cosine form with the phase shift $-\pi \le \delta \le \pi$. When written in this form, the argument of the cosine is the phase of the oscillatory factor. Set this equal to π to find the approximate value of t where that cosine factor reaches its first minimum value of -1 where the transient will touch the lower envelope curve. Mark this on your plot in part d).

d) Plot the transient with its envelope curves (state them) together for 5 characteristic times of the decaying exponential factor. [plot 1]

e) Plot the full solution together with the transient for $0 \le t \le 6$. Does your plot agree with the approximate value of the steady state amplitude? Explain. [plot 2]

2. Investigate resonance for the following differential equation $9x'' + 6x' + 37x = \cos(\omega t)$. First evaluate the parameters $k_0, \tau_0, \omega_0, B_0$ exactly and then Q approximately (2 decimal places) and then use calculus plus the formula for the response amplitude

$$A(\omega) = B_0 \cdot \left(\left(\omega^2 - \omega_0^2\right)^2 + k_0^2\,\omega^2 \right)^{-\frac{1}{2}},$$

to find the exact and 4 decimal place approximations for the peak frequence ω_{peak} and amplitude $A(\omega_{peak})$ and do the same for the natural freqency ω_0 and amplitude. [Be sure to show your calculus steps and evaluation details by hand.]

Compare the value of the ratio $\dfrac{A(\omega_{peak})}{A(0)}$ with Q. Finally evaluate and plot the ratio $\dfrac{A(\omega)}{A(0)}$ versus the frequency for $0 \le \omega \le 10$ with the gridline option on. [plot 3]

3. $x_1' = -4x_1 - x_2 - x_3, x_2' = -x_1 - 4x_2 - x_3, x_3' = -x_1 - x_2 - 4x_3, x_1(0) = 3, x_2(0) = -6, x_3(0) = 9$

a) Write this DE system and its initial conditions in matrix form, identifying its coefficient matrix.

b) Write down the determinant condition leading to its characteristic equation, using Maple to evaluate that

determinant and solving the characteristic equation.
c) For each eigenvalue, by hand find basis vectors for its eigenspace using Maple only for the row reduction if necessary (the reduction by hand is quick!) carefully documenting each step clearly, and assemble these eigenvectors into the matrix *B*, correlating the columns with the eigenvalues.
d) Evaluate the diagonalized coefficient matrix.
e) Re-express the matrix DE system in terms of the new vector variable $y = B^{-1} x$ to decouple them. Write down the three uncoupled scalar DEs and solve them. Then transform back to the original variables to get the general solution.
f) Use the inverse matrix to solve the initial conditions and write down the three scalar solutions of the initial value problem.
g) Plot these three functions together for 5 characteristic times of the longest exponential characteristic time in the solution. What is this characteristic time? [plot 4]

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Plot multiple expressions together:

> $plot\left(\left[\cos(4\,t), e^{-t}\cdot(1+\sin(4\,t)), e^{-t}\right], t=0..5, color=[red, blue, black], gridlines=true\right)$

or to specify the vertical window different from Maple's choice:

> $plot\left(\left[\cos(4\,t), e^{-t}\cdot(1+\sin(4\,t)), e^{-t}\right], t=0..5, -2..2, color=[red, blue, black], gridlines=true\right)$

▶ solution