MAT2705-04/05 23S Test 2

① a) $$\begin{aligned} x_1+3x_2-4x_3-8x_4&=6\\ x_1 \qquad +2x_3+x_4&=3\\ 2x_1+7x_2-10x_3-19x_4&=13 \end{aligned}$$
(scalar form)

$$\rightarrow \underbrace{\begin{bmatrix} 1 & 3 & -4 & -8\\ 1 & 0 & 2 & 1\\ 2 & 7 & -10 & -19 \end{bmatrix}}_{A} \begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 6\\ 3\\ 13 \end{bmatrix}}_{b}$$ (matrix form)

b) augmented matrix:

$$\langle A \mid b\rangle = \begin{bmatrix} 1 & 3 & -4 & -8 & 6\\ 1 & 0 & 2 & 1 & 3\\ 2 & 7 & -10 & -19 & 13 \end{bmatrix}$$

$\downarrow$ rref by Maple

$$\begin{bmatrix} 1 & 0 & 2 & 1 & 3\\ 0 & 1 & -2 & -3 & 1\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c) $$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4\\ L & L & \underbrace{F \quad F} \end{array}$$

$x_3=t_1,\ x_4=t_2$

leading: x_1, x_2

free: x_3, x_4

d) $$\begin{aligned} x_1 \quad +2x_3+x_4 &= 3 &\rightarrow\ & x_1=3-2t_1-t_2\\ x_2-2x_3-3x_4 &= 1 &\rightarrow\ & x_2=1+2t_1+3t_2\\ 0&=0 \end{aligned}$$

$$\begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4 \end{bmatrix} = \begin{bmatrix} 3-2t_1-t_2\\ 1+2t_1+3t_2\\ t_1\\ t_2 \end{bmatrix}$$

e) $$= \underbrace{\begin{bmatrix} 3\\ 1\\ 0\\ 0 \end{bmatrix}}_{x_p} + \underbrace{\begin{bmatrix} -2t_1-t_2\\ 2t_1+3t_2\\ t_1\\ t_2 \end{bmatrix}}$$

f) $$x_h = t_1\begin{bmatrix} -2\\ 2\\ 1\\ 0 \end{bmatrix} + t_2\begin{bmatrix} -1\\ 3\\ 0\\ 1 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} -2\\ 2\\ 1\\ 0 \end{bmatrix}, \begin{bmatrix} -1\\ 3\\ 0\\ 1 \end{bmatrix} \right\}$$ is a basis of the soln space

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② a) $v_1 = \begin{bmatrix}1\\1\\-1\\1\end{bmatrix}, v_2 = \begin{bmatrix}2\\1\\1\\1\end{bmatrix}, v_3 = \begin{bmatrix}3\\1\\4\\2\end{bmatrix}$ $\quad \langle v_1|v_2|v_3\rangle = \begin{bmatrix}1&2&3\\1&1&1\\-1&1&4\\1&1&2\end{bmatrix}$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0 \to \begin{bmatrix}1&2&3\\1&1&1\\-1&1&4\\1&1&2\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

(test for linear independence)

$\downarrow$ rref (Maple)

$$\begin{array}{c}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}\\ \begin{matrix}c_1&c_2&c_3\\L&L&L\end{matrix}\end{array}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \xrightarrow{\text{reduced eqns}} \left.\begin{matrix}c_1=0\\c_2=0\\c_3=0\\0=0\end{matrix}\right\}$$

no linear relationships exist (nontrivial ones?)

$\boxed{\text{These are linearly independent.}}$

b) $v_1 = \begin{bmatrix}3\\0\\1\\2\end{bmatrix}, v_2 = \begin{bmatrix}1\\-1\\0\\1\end{bmatrix}, v_3 = \begin{bmatrix}1\\2\\1\\0\end{bmatrix}$ $\quad \langle v_1|v_2|v_3\rangle = \begin{bmatrix}3&1&1\\0&-1&2\\1&0&1\\2&1&0\end{bmatrix}$

again: $\begin{bmatrix}3&1&1\\0&-1&2\\1&0&1\\2&1&0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

$\downarrow$ rref (Maple)

$$\begin{array}{c}\begin{bmatrix}1&0&-1\\0&1&2\\0&0&0\\0&0&0\end{bmatrix}\\ \begin{matrix}c_1&c_2&c_3\\L&L&F\end{matrix}\end{array}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \xrightarrow{\text{reduced eqns}} \begin{matrix}c_1 + c_3 = 0 \to c_1 = -t\\ c_2 - 2c_3 = 0 \to c_2 = 2t\\ 0=0\end{matrix}$$

$\hookrightarrow c_3 = t$

$\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}-t\\2t\\t\end{bmatrix} = t\begin{bmatrix}-1\\2\\1\end{bmatrix}$

$\uparrow$ desired coeffs!

$\boxed{\text{Nonzero soln means they are linearly dependent.}} \longrightarrow \boxed{-v_1 + 2v_2 + v_3 = 0}$ simplest linear relationship

(indeed: $-\begin{bmatrix}3\\0\\1\\2\end{bmatrix} + 2\begin{bmatrix}1\\-1\\0\\1\end{bmatrix} + \begin{bmatrix}1\\2\\1\\0\end{bmatrix} = \begin{bmatrix}-3+2+1\\0-2+2\\\\-2+2+0\end{bmatrix} = \begin{bmatrix}0\\0\\\\0\end{bmatrix}$ ✓)

(3)
$$\begin{aligned} 3x_1 - 4x_2 + 2x_3 &= 0 \\ -2x_1 \qquad + 3x_3 &= -7 \\ x_1 + x_2 - 3x_3 &= 6 \end{aligned}$$

$$\begin{bmatrix} 3 & -4 & 2 \\ -2 & 0 & 3 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 0 & 3 \\ 1 & 1 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -7 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 & 10 & 12 \\ 3 & 11 & 13 \\ 2 & 7 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ -7 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 70 + 72 \\ 0 - 77 + 78 \\ 0 - 49 + 48 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

so $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ is the soln.