

MAT1705-04/05 235 Test 1

① a) $\frac{dy}{dx} = \frac{e^{x/2}}{3y^2}$ separable

$\int y^2 dy = \int \frac{1}{3} e^{x/2} dx$ separate & integrate

$\frac{1}{3} y^3 = \frac{1}{3} \frac{e^{x/2}}{1/2} + C_1 = \frac{2}{3} e^{x/2} + C_1$

$y^3 = 3(\frac{2}{3} e^{x/2} + C_1) = 2e^{x/2} + \frac{3C_1}{\equiv C}$

$y = (2e^{x/2} + c)^{1/3}$ gen soln

b) $2 = y(0) = (2e^0 + c)^{1/3} = (2+c)^{1/3}$

$8 = 2^3 = 2+c \rightarrow c = 8-2 = 6$

$y = (2e^{x/2} + 6)^{1/3}$

c) $\frac{dy}{dx} = \frac{1}{3} (2e^{x/2} + 6)^{-2/3} (2e^{x/2} \cdot \frac{1}{2} + 0)$
 $= \frac{1}{3} \frac{e^{x/2}}{(2e^{x/2} + 6)^{2/3}}$

$\frac{e^{x/2}}{3y^2} = \frac{1}{3} \frac{e^{x/2}}{((2e^{x/2} + 6)^{1/3})^2} = \frac{1}{3} \frac{e^{x/2}}{(2e^{x/2} + 6)^{2/3}}$ LHS } RHS ✓

MAT2705-04/05 Test 1 (2)

② a) $16 \frac{dx}{dt} = 4 - x \Rightarrow 0 \rightarrow \boxed{x=4}$ is a constant soln

b) $16 \frac{dx}{dt} + x = 4$

$e^{\frac{t}{16}} \left[\frac{dx}{dt} + \frac{1}{16}x = \frac{4}{16} = \frac{1}{4} \right] \rightarrow$
 $\int \frac{1}{16} dt = \frac{t}{16}$ integrating factor

$e^{\frac{t}{16}} \left(\frac{dx}{dt} + \frac{x}{16} \right) = \frac{1}{4} e^{\frac{t}{16}}$
 $\frac{d}{dt} (x e^{\frac{t}{16}}) = \frac{1}{4} e^{\frac{t}{16}}$
 $x e^{\frac{t}{16}} = \int \frac{1}{4} e^{\frac{t}{16}} dt$
 $= \frac{1}{4} e^{\frac{t}{16}} + C$
 $= 4 e^{\frac{t}{16}} + C$

$x = e^{-\frac{t}{16}} (4e^{\frac{t}{16}} + C)$

$\boxed{x = 4 + C e^{-t/16}}$

c) $20 = x(0) = 4 + C e^0 \rightarrow C = 20 - 4 = 16$

$\boxed{x = 4 + 16 e^{-t/16}}$

d) $\lim_{t \rightarrow \infty} x = \lim_{t \rightarrow \infty} (4 + 16 e^{-t/16}) = \boxed{4}$ agrees with part a)

e) $8 = 4 + 16 e^{-t/16}$

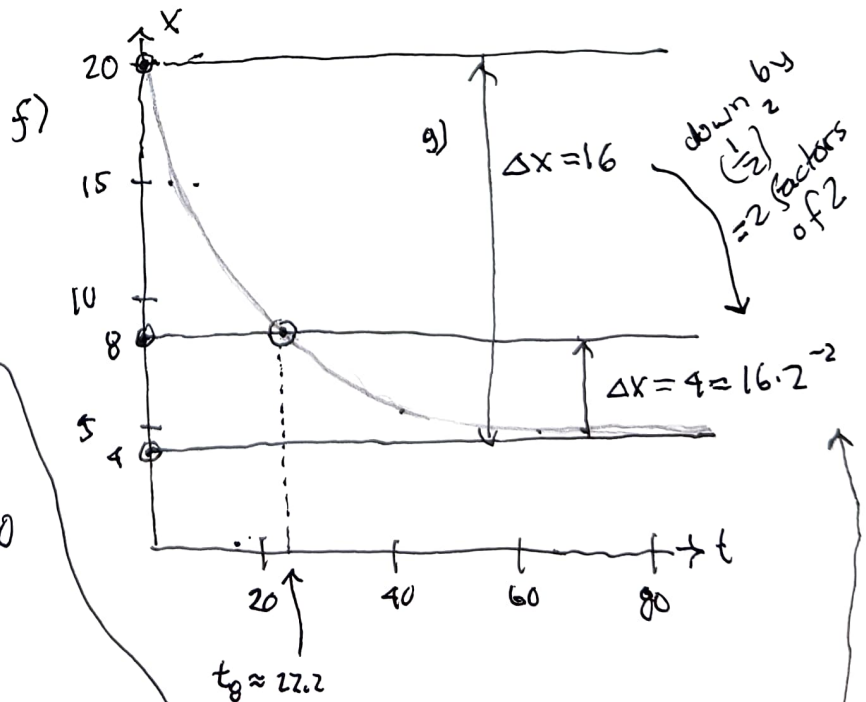
$4 = 16 e^{-t/16}$

$e^{t/16} = \frac{16}{4} = 4$

$\frac{t}{16} = \ln 4$

$\boxed{t = 16 \ln 4 \approx 22.18}$

f) $e^{-t/16} \rightarrow \tau = 16$
 $5\tau = 5 \cdot 16 = 80$



optional

g) half-life $e^{-\frac{t}{16}} = \frac{1}{2}$

$e^{\frac{t}{16}} = 2 \quad \frac{t}{16} = \ln 2 \quad t = 16 \ln 2$
 $= \frac{1}{2} \ln(\ln 2^2) = \frac{1}{2} (16 \ln 4)$

so $t_g = 2$ half, difference decreases by 2 factors of 2