Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). **Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.**

▼ **pledge**

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Consider the initial value problem $\dfrac{dy}{dx} = \dfrac{e^{\frac{x}{2}}}{3\,y^2}, y(0) = 2$.

a) Find the general solution, step by step.
b) Find the initial value solution.
c) Show by direct substitution and independent simplification of each side of the differential equation that you answer to part b) does satisfy that equation.

2. A mixing tank problem is described by the initial value problem
$$16\,\frac{dx}{dt} = 4 - x,\, x(0) = 20\,,$$
where $x(t)$ is the amount of solute in the solution as a function of the time t.
a) What is the equilibrium solution of this differential equation?
b) Use the linear solution technique (not the separable approach!) to derive the general solution to this problem step by step.
c) Impose the initial value condition on this solution to solve the initial value problem for $x(t)$.
d) What is the asymptotic value x_∞ of $x(t)$ for large t (i.e., $x_\infty = \lim\limits_{t \to \infty} x(t)$)?
e) How long will it take for the solute level to reduce to $x = 8$? Give an exact solution in terms of a "ln" expression and its decimal value to 2 decimal places.
f) What is the value of the characteristic time τ for the decaying exponential in the solution? Then make a rough sketch of a technology plot of this solution for $t = 0 ..5\,\tau$, and include the asymptote $x = x_\infty$ in the diagram and the point where $x = 8$, indicating the corresponding time on the horizontal axis. Be sure to put adequate tickmarks on both axes.
g) **Optional.** How is your value for part e) related to the half-life for this decaying exponential? Why does this make sense?

▶ **solution**

Do not write on the test sheet other than your name, signature and date.