

MAT 1505-02/03 23F Quiz 9

① a)  $f(x) = \sin x$      $f(0) = 0$   
 $f'(x) = \cos x$      $f'(0) = 1$   
 $f''(x) = -\sin x$      $f''(0) = 0$   
 $f^{(3)}(x) = -\cos x$      $f^{(3)}(0) = -1$   
 $f^{(4)}(x) = \sin x$      $f^{(4)}(0) = 0$   
 $f^{(5)}(x) = \cos x$      $f^{(5)}(0) = 1$

$$T_5(x) = \sum_{n=0}^5 \frac{f^{(n)}(0)}{n!} x^n = \frac{1}{1!}x^1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 = \boxed{x - \frac{1}{6}x^3 + \frac{1}{120}x^5}$$

b)  $\frac{|x|^5}{120} = .001 \rightarrow |x|^5 = (.001)(120) \rightarrow |x| = [(.001)(120)]^{1/5} = \boxed{0.654}$   
 $0.654 * \frac{180}{\pi} = \boxed{37.5^\circ} > 30^\circ!$

c)  $\frac{\pi}{6} = \boxed{0.5236} < 0.654$

d)  $T_3(\frac{\pi}{6}) = \frac{\pi}{6} - \frac{1}{6}(\frac{\pi}{6})^3 = 0.49967 \approx \boxed{0.4997}$

e)  $\sin \frac{\pi}{6} = \frac{1}{2} = 0.5$

$|\sin \frac{\pi}{6} - T_3(\frac{\pi}{6})| = 0.50000 - 0.4997 = \boxed{.0003} < .001!$

② a)  $\sin \frac{\pi}{6} = \frac{1}{2}$ ,  $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$      $\frac{\pi}{6} = 30^\circ$

$f(\frac{\pi}{6}) = \sin \frac{\pi}{6} = \frac{1}{2}$

$f'(\frac{\pi}{6}) = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$f^{(2)}(\frac{\pi}{6}) = -\sin \frac{\pi}{6} = -\frac{1}{2}$

$f^{(3)}(\frac{\pi}{6}) = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$$T_3(x) = \sum_{n=0}^3 \frac{f^{(n)}(\frac{\pi}{6})}{n!} (x - \frac{\pi}{6})^n = \frac{1}{2} + \frac{\sqrt{3}}{2} (x - \frac{\pi}{6}) - \frac{1}{2} \frac{(x - \frac{\pi}{6})^2}{2!} - \frac{\sqrt{3}}{2} \frac{(x - \frac{\pi}{6})^3}{3!}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} (x - \frac{\pi}{6}) - \frac{1}{4} (x - \frac{\pi}{6})^2 - \frac{\sqrt{3}}{12} (x - \frac{\pi}{6})^3$$

b)  $28^\circ \approx \frac{28\pi}{180} = \boxed{\frac{7\pi}{45} = x}$  but  $28^\circ - 30^\circ = -2^\circ = -2 \frac{\pi}{90} = \boxed{-\frac{\pi}{90} = x - \frac{\pi}{6}}$

$T_3(\frac{7\pi}{45}) = \frac{1}{2} + \frac{\sqrt{3}}{2} (-\frac{\pi}{90}) - \frac{1}{4} (-\frac{\pi}{90})^2 - \frac{\sqrt{3}}{12} (-\frac{\pi}{90})^3 \stackrel{\text{Maple}}{=} 0.4694715323$

c)  $\sin(\frac{7\pi}{45}) = 0.4694715630$

last 3 differ but rounding to 7 digits leads to differing final digits  
 $\approx 0.469472$  for both to 6 digits.