MAT1505-02/03 23F Quiz 9 Print Name (Last, First) _____ , _____|____
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.** Numerical values can be evaluated with technology.

1. As in the video at the beginning of section 11.11 for $\cos(x)$, consider the (*n*th degree) Taylor polyomials for $f(x) = \sin(x)$ centered at $a = 0$: $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$.

a) Use this Taylor formula to derive the fifth degree Taylor polynomial $T_5(x)$.

b) Because this is an alternating odd power series, the last term in absolute value is the maximum error in using the third degree Taylor polynomial to approximate $\sin(x)$. For what values of $|x|$ is this fifth power term less than 0.001? What angle in degrees corresponds to this angle in radians (to the nearest degree)?

c) Evaluate numerically $\frac{\pi}{6}$ confirming that a 30 degree angle lies within this interval you found for $|x|$.

d) Evaluate $T_3\left(\frac{\pi}{6}\right)$ numerically to 4 decimal places.

e) Confirm that the actual error $\left|\sin\left(\frac{\pi}{6}\right) - T_3\left(\frac{\pi}{6}\right)\right| < 0.001$.

2. a) Following example 7 of section 11.10, evaluate the third degree Taylor polynomial of $f(x) = \sin(x)$ centered at $a = \frac{\pi}{6}$ using the Taylor formula.

b) What are the exact values of the angle x and of $x - \frac{\pi}{6}$ in radians for the angle 28°? [Simplify the coefficients of π.]

c) Use this Taylor polynomial to approximate the $\sin(28°) = \sin(x)$. Give the result to 10 decimal places.

d) Evaluate the numerical value of $\sin(28°)$ to 10 decimal places using technology. Confirm that they agree to 6 decimal places when rounding off to 6 decimal places. [Note that rounding off to 7 decimal places gives differing final digits!]

▶ solution