MAT1505-02/03 23F Quiz 8 Print Name (Last, First) ___________________ , ________________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.** Numerical values can be evaluated with technology.

1. Consider the definite integral $\int_0^{0.2} \frac{x}{1+x^3}\,dx$ and use a power series representation of the integrand to approximate it to 6 decimal places by following the steps:

a) Use the geometric series power series trick to express $\frac{x}{1+x^3}$ as a power series, stating the sigma formula involving the simplified nth term in the series.

b) Give the power series which results from term by term integration $\int \frac{x}{1+x^3}\,dx$ setting the constant of integration to zero to get the antiderivative which vanishes at $x = 0$

c) Write out explicitly the first four nonzero terms in this indefinite integral series.

d) Use it to evaluate the truncated definite integral power series consisting of the first four terms without simplifying the powers of 0.2, so that you can evaluate these terms before combining any number of them below.

e) Evaluate the numerical value of each of these 4 terms. [Even a calculator will do here.]

f) Using the fact that this is an alternating series, how many terms should be needed to get that 6 decimal place accuracy according to the alternating series estimate for the maximum absolute value of the truncation error?

g) Evaluate the partial sum for this number of terms, as well as for the next partial sum to see if that next term affects round off error.

h) State your final value to 6 decimal places based on part g). It should agree to 6 decimal places with Maple's value for this definite integral. Does it?

▶ solution

MAT 1505-02/03 23F Quiz 8

1\. a) $\frac{x}{1+x^3} = x\cdot\frac{1}{1-(-x^3)} = x\sum_{n=0}^{\infty}(-x^3)^n = \sum_{n=0}^{\infty}(-1)^n x^{3n}\cdot x = \boxed{\sum_{n=0}^{\infty}(-1)^n x^{3n+1}}$

$= x - x^4 + x^7 - x^{10} + \cdots$

b) $\int \frac{x}{1+x^3}\,dx = \int \sum_{n=0}^{\infty}(-1)^n x^{3n+1}\,dx = \sum_{n=0}^{\infty}(-1)^n \int x^{3n+1}\,dx = \boxed{\sum_{n=0}^{\infty}(-1)^n \frac{x^{3n+2}}{3n+2} + \underbrace{C}_{0}}$

c) $= \frac{x^2}{2} - \frac{x^5}{5} + \frac{x^8}{8} - \frac{x^{11}}{11} + \cdots$

d) $\int_0^{0.2} \frac{x}{1+x^3}\,dx = \left[\frac{x^2}{2} - \frac{x^5}{5} + \frac{x^8}{8} - \frac{x^{11}}{11} + \cdots\right]\Big|_0^{0.2}$ $0 \to$ no contribution (zero)

$= \frac{0.2^2}{2} - \frac{0.2^5}{5} + \frac{0.2^8}{8} - \frac{0.2^{11}}{11} + \cdots$

e) $= \underbrace{\underset{n=0}{0.02} - \underset{n=1}{0.000064}}_{\text{sum of first two terms:}} + \underbrace{3.2\cdot 10^{-7} - 1.86\cdot 10^{-9} + \cdots}$

alternating series estimate

f) error in truncating after first 2 terms $< 3.2\cdot 10^{-7}$ (next term) $\leftarrow$ affects 7th decimal place

so $n=0,1$ are summed in theory

g) $0.\underbrace{019936}0$

sum of first three terms (7 decimal places):

$$\begin{array}{r} .0199360 \\ +.0000003 \\ \hline .0199363 \end{array} \approx \boxed{.019936}$$

h)

still rounds off to same 6th decimal place

Maple agrees!