MAT 1505-02/03 23F Quiz 6

① a) $\int_0^a c\,(a-x)x^2dx = c\int_0^a ax^2-x^3dx = c\left(a\frac{x^3}{3}-\frac{x^4}{4}\right)\Big|_0^a = c\left(\frac{a^4}{3}-\frac{a^4}{4}\right)-0$

$= ca^4\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{c}{12}a^4 = 1 \rightarrow \boxed{c=\frac{12}{a^4}} \rightarrow p(x)=\frac{12}{a^4}(a-x)\,x^2$

b) $\mu_x = \int_0^a x\cdot\frac{12}{a^4}(a-x)x^2dx = \frac{12}{a^4}\int_0^a ax^3-x^4dx = \frac{12}{a^4}\left(a\frac{x^4}{4}-\frac{x^5}{5}\right)\Big|_0^a$

$= \frac{12}{a^4}a^5\left(\frac{1}{4}-\frac{1}{5}\right) = \frac{12}{20}a = \boxed{\frac{3}{5}a}$

c) $\int_{x_1}^{x_2}\frac{12}{a^4}(a-x)x^2dx = \int_{u_1}^{u_2}\frac{12}{a^4}(a-au)(au)^2(a\,du)$

$\left[\begin{array}{l} u=\frac{x}{a}\rightarrow x=au \\ \quad dx=a\,du \\ x=x_1\rightarrow u=x_1/a \\ x=x_2\rightarrow u=x_2/a \end{array}\right]$

$= \int_{u_1}^{u_2}\frac{12}{a^4}a^4(1-u)u^2du = \int_{u_1}^{u_2}12(1-u)u^2du$ ✓

d) $p_s(u) = 12(1-u)u^2 = 12(u^2-u^3) > 0$ on $(0,1)$ must have max there

$p_s'(u) = 12(2u-3u^2) = 12(2-3u)u = 0 \rightarrow u=0, \frac{2}{3}$

so $\boxed{u_p=\frac{2}{3}}\boxed{\approx 0.667}$

e) $\mu = \frac{\mu_x}{a} = \frac{\frac{3}{5}a}{a} = \boxed{\frac{3}{5}=0.600}$

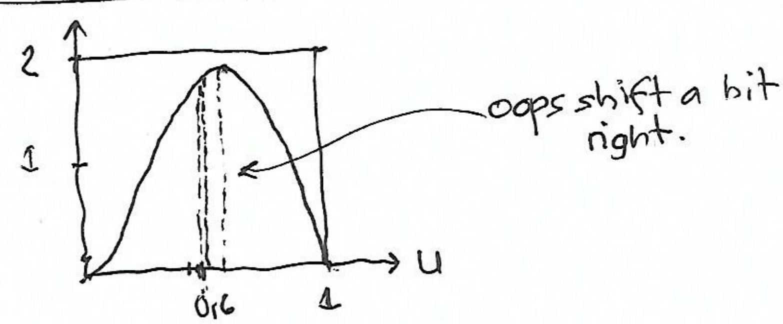
f) $\int_0^{u_m}12(1-u)u^2du = \int_0^{u_m}12(u-u^3)\,du = 12\left(\frac{u^2}{2}-\frac{u^4}{4}\right)\Big|_0^{u_m}$

$= 6u_m^2-3u_m^4 = \frac{1}{2} \xrightarrow[\text{fsolve}]{} \underbrace{0.6143}, 1.2475$

$\boxed{u_m\approx 0.614}$

g) $\boxed{\mu=0.600 < u_m\approx 0.614 < u_p\approx 0.667}$ all pretty close

see Maple for graph.



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(2) $p(x) = \frac{1}{\sqrt{2\pi\cdot 15^2}}\, e^{-\frac{(x-100)^2}{2\cdot 15^2}} = \frac{1}{15\sqrt{2\pi}}\, e$

$> \text{evalf}\left(\int_{130}^{\infty} p(x)\,dx \right) \approx 0.022750 \approx 0.023 \rightsquigarrow \boxed{2.3\,\%}$

$> \text{evalf}\left(\int_{140}^{\infty} p(x)\,dx \right) \approx 0.003830 \approx 0.0038 \rightsquigarrow \boxed{0.4\,\%}$

$0.022750 * 332 \approx 7.5530 \approx \boxed{7.6 \text{ million}}$

$0.003830 * 332 \approx 1.27169 \approx \boxed{1.3 \text{ million}}$

always keep many more significant figures for intermediate numbers

in this case it turns out not to make a difference