MAT1505-02/03 23F Quiz 5   Print Name (Last, First) ____________________ , _________________|_____

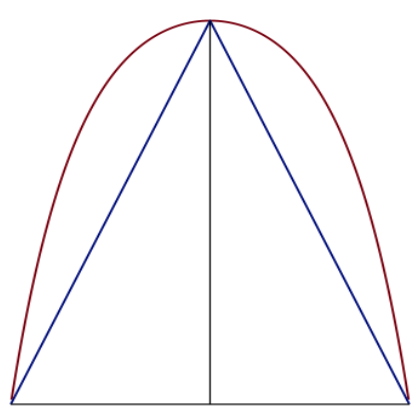
Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.**

1. a)

50. The British physicist and architect Robert Hooke (1635–1703) was the first to observe that the ideal shape for a free standing arch is an inverted catenary. Hooke remarked, "As hangs the chain, so stands the arch." The Gateway Arch in St. Louis is based on the shape of a catenary; the central curve of the arch is modeled by the equation

$$y = 211.49 - 20.96 \cosh 0.03291765x$$

where x and y are measured in meters and $|x| \leqslant 91.20$. Set up an integral for the length of the arch and evaluate the integral numerically to estimate the length correct to the nearest meter.



b) For comparison, evaluate the length of the two sides of the inscribed triangle from the endpoints shown in the figure, to the nearest meter.
c) How much bigger in percent is the arch length compared to the triangle approximation length? Round off to the nearest percent.
d) What is the height of the arch in feet to the nearest foot according to this model? (The conversion factor is 1 m = 3.28084 *ft* •)
e) Search for the height of the arch, how do you explain the small difference from the value you calculated?

2. a) The integral $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{x}\cdot(1+x)}\,\mathrm{d}x$ is improper because of both endpoints, so separate it into the sum of the integrals up to 1 and from 1 out to infinity. Then write out the limits which define these improper integrals.
b) To evaluate these limits, you need the antiderivative of the integrand, which you can find by a change of variable $u = \sqrt{x}$ in the indefinite integral $\displaystyle\int \frac{1}{\sqrt{x}\cdot(1+x)}\,\mathrm{d}x$. What is the value of this indefinite integral?
c) Now use this in each of the limits to separately evaluate them and then combine them to the final value of the original improper integral.
d) Notice that the integrand looks like $x^{-\frac{1}{2}}$ at the origin (where $1+x\to 1$) which integrates to $2\,x^{\frac{1}{2}}$, while for very large x it looks like $x^{-\frac{3}{2}}$ which integrates to $-2\,x^{-\frac{1}{2}}$. which means in both cases the antiderivative is leading to convergence at those endpoints, which explains why the integral converges, ignoring the details. Does this make sense to you?

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① a) $y = 211.49 - 20.96\cosh(0.03291765x)$

$$\frac{dy}{dx} = 0 - \underbrace{20.96(0.03291765)}_{0.6899539440}\sinh(0.03291765x)$$

$$\left(\frac{dy}{dx}\right)^2 = 0.4760364448\sinh^2(0.03291765x)$$

$$L = \int_{-91.20}^{91.20}\sqrt{1 + 0.4760364448\sinh^2(0.03291765x)}\,dx$$

$$\underset{\text{Maple}}{=} 451.1370409 \approx \boxed{451}\ \text{m}$$

b) $H = 211.49 - 20.96\underbrace{\cosh(0)}_{1} = 190.53$

$W = 91.20$

$$L_{approx} = 2\sqrt{W^2+H^2} \approx 422.464770 \approx \boxed{422}\ \text{m}$$

c) $$\frac{L}{L_{approx}} = \frac{451.137}{422.464} \approx 1.067869017 \approx 1.07$$

so L is about $\boxed{7\%}$ larger than L_{approx}

d) $H\,\text{m}\cdot(3.28084\ \text{ft/m}) \approx 625.0984452\ \text{ft} \approx \boxed{625\ \text{ft}}$

e) The height is given at 630 ft which probably is the highest point on the arch, but it has a vertical thickness at the top of at least 10 ft if not more, so the mathematical curve must be inside the physical arch. In fact the problem statement refers to the "central curve" of what is an equilateral triangle cross-section.

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② a) $\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx = \int_0^1 \frac{1}{x^{1/2}(1+x)}\,dx + \int_1^\infty \frac{1}{x^{1/2}(1+x)}\,dx$

$= \lim_{t\to 0^+} \int_t^1 \frac{1}{x^{1/2}(1+x)}\,dx + \lim_{t\to\infty} \int_1^t \frac{1}{x^{1/2}(1+x)}\,dx$

b) $\int \frac{1}{(1+x)}\overbrace{\frac{dx}{x^{1/2}}}^{2\,du} = \int \frac{1}{1+u^2}\cdot 2\,du = 2\int \frac{1}{1+u^2}\,du = 2\arctan u + C$

$= \boxed{2\arctan x^{1/2} + C}$

$u = x^{1/2} \to x = u^2$

$du = \frac{1}{2}x^{-1/2}\,dx$

$= \frac{1}{2x^{1/2}}\,dx$

$2\,du = \frac{dx}{x^{1/2}}$

c) $\lim_{t\to 0^+} \int_t^1 \frac{1}{x^{1/2}(1+x)}\,dx = \lim_{t\to 0^+} 2\arctan x^{1/2}\Big|_t^1$

$= \lim_{t\to 0^+}\left(2\underbrace{\arctan 1}_{\frac{\pi}{4}} - 2\arctan t \xrightarrow{} 0\right) = \boxed{\frac{\pi}{2}}$

$\lim_{t\to\infty} \int_1^t \frac{1}{x^{1/2}(1+x)}\,dx = \lim_{t\to\infty} 2\arctan x^{1/2}\Big|_1^t$

$= \lim_{t\to\infty}\left(2\underbrace{\arctan t^{1/2}}_{\frac{\pi}{2}} - 2\underbrace{\arctan 1}_{\frac{\pi}{4}}\right) = 2\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{4}\right)$

$= \pi - \frac{\pi}{2} = \boxed{\frac{\pi}{2}}$

so $\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx = \frac{\pi}{2} + \frac{\pi}{2} = \boxed{\pi}$

d) This reasoning makes sense to bob. I hope it does to you as well, since intuitive understanding trumps blind evaluation in terms of developing mathematical ability.