MAT1505-02/03 23F Quiz 4 Print Name (Last, First) _____ , _____|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.**

1. a) A cup of coffee has temperature $95°\,C$ and takes 60 minutes to cool to $42°\,C$ in a room with temperature $20°\,C$. According to Newton's Law of Cooling, the temperature of the coffee after t minutes is $T(t) = 20 + 75\,e^{-kt}$, where $k \approx 0.02$. What is the average temperature T_{avg} of the coffee during this first hour?

b) Make a rough sketch of a plot window $0 \le t \le 60, 0 \le T \le 100$ showing both T and T_{avg} (or print out a Maple plot with hand annotations). Does this average value look right (area above equals area below?)

2. a) How much work does it take to build a conical "pyramid" from the ground up if the radius of the base is r and its height is h using a material which has a weight density of ρ ? Make a diagram illustrating your work in lifting a typical horizontal plane cross-section into place from the ground.

b) If the total weight is $w = \rho V = \rho \cdot \left(\frac{1}{3}\pi r^2 h\right)$, to what height must the entire weight be raised at once to do an equivalent amount of work?

c) **Optional: practice parsing words in an application in using a calculus word problem result.** Dividing the work done by the height of the pyramid gives the average amount of work done per unit height (units are work/length) to raise a layer of material to each final position (this is how one can interpret the integrand of the work integral). What fraction of the total weight does this represent?

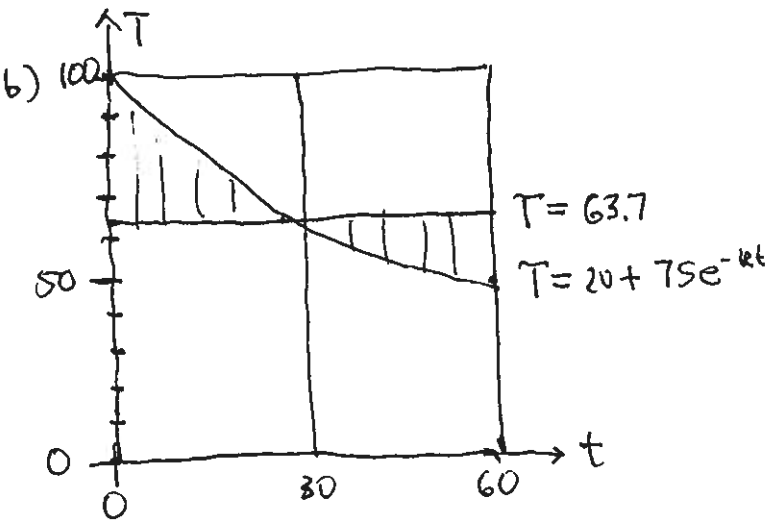
▶ solution

① a) $T_{avg} = \frac{1}{60}\int_0^{60} 20 + 75e^{-kt}\,dt = \frac{1}{60}\left(20t + \frac{75}{-k}e^{-kt}\right)\Big|_0^{60}$

$= \frac{1}{60}\left[60(20) - \frac{75}{k}(e^{-k60} - 1)\right] \approx$ Maple $63.675 \approx \boxed{63.7°}$

The numbers stated have at most 2 significant figures, so keep at most one more in the final answer

b)



okay, my sketch is a bit off but on my screen the two areas above and below the average line look about equal as they should so "it looks right".

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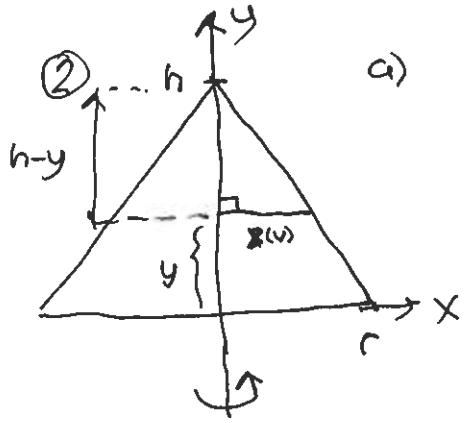

cone = volume of revolution

a) $\frac{X(y)}{h-y} = \frac{r}{h} \rightarrow X(y) = \frac{r}{h}(h-y)$

$A(y) = \pi X(y)^2$

$W = \int_0^h y \underbrace{\rho A(y)\, dy}_{dw = \text{differential of weight}}$

$= \int_0^h y\rho \cdot \pi \left(\frac{r}{h}(h-y)\right)^2 dy$

$= \frac{\pi\rho r^2}{h^2} \int_0^h \underbrace{y(h-y)^2}_{y(h^2-2hy+y^2) = yh^2-2hy^2+y^3} dy$

$= \frac{\pi\rho r^2}{h^2} \int_0^h yh^2 - 2hy^2 + y^3\, dy = \frac{\pi\rho r^2}{h^2}\left(\frac{y^2h^2}{2} - \frac{2hy^3}{3} + \frac{y^4}{4}\right)\Big|_0^h$

$= \frac{\pi\rho r^2}{h^2}\left(\frac{h^4}{2} - \frac{2}{3}h^4 + \frac{h^4}{4}\right) = \pi\rho h^2 r^2\left(\frac{6}{12} - \frac{8}{12} + \frac{3}{12}\right) = \boxed{\frac{1}{12}\pi\rho h^2 r^2}$

b) $w = \frac{\pi}{3}\rho r^2 h$

$\frac{W}{w} = \frac{\frac{1}{12}\pi\rho h^2 r^2}{\frac{1}{3}\pi\rho r^2 h} = \boxed{\frac{h}{4}}$ so $W = \underbrace{\left(\frac{h}{4}\right)w}$

work done lifting w up height h/4.

c) $\frac{W}{h} = \frac{1}{h}\int_0^h \underbrace{y\rho A(y)}_{\text{work done to raise the layer at } y \text{ of thickness } dy}\, dy$

$\underbrace{\text{work done}}_{\text{unit length}}$ to raise the layer at y of thickness "dy"

product with dy = work done to raise that layer

but who cares! we just want to evaluate this ratio

$= \frac{\frac{1}{12}\pi\rho h^2 r^2}{h} = \frac{1}{12}\pi\rho r^2 h = \frac{1}{4}w$ just 1/4 the total weight

$\boxed{\frac{1}{4}}$