MAT1505-02/03 23F Quiz 3   Print Name (Last, First) ______________________ , ______________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.**

1. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the x-axis are squares with one side lying along the base. Make a 2d diagram of the region of integration in the x-y plane, completely labeled by relevant equations and tickmarks. If you are feeling ambitious, make your best shot at a 3d diagram of this solid with a typical square plane cross-section indicated. Be sure you justify each step even though the results might seem obvious.

2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the 2d region, the solid, and a typical washer cross-section, labeling everything clearly, including the two typical radii:

$y = \sqrt{x-1}$, $y = 0$, $x = 5$ about the axis $y = -1$.

▸ solution

① a) $y = x^2$, $y = 2 - x^2$ $\rightarrow x^2 = 2 - x^2 \rightarrow 2x^2 = 2 \rightarrow x^2 = 1 \rightarrow x = \pm 1$ intersection pts.

$S(x) = (2 - x^2) - x^2 = 2(1 - x^2)$ base side length of square

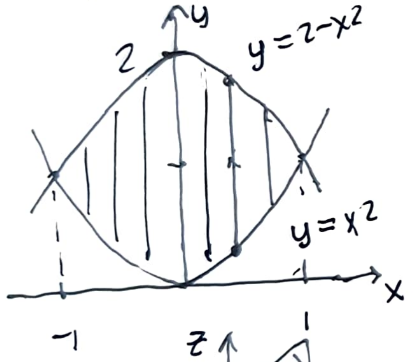
$A(x) = S(x)^2 = (2(1-x^2))^2 = 4(1-x^2)^2 = 4(1 - 2x^2 + x^4)$

$$V = \int_{-1}^{1} A(x)\,dx = 2\int_0^1 A(x)\,dx = \boxed{2\int_0^1 4(1-2x^2+x^4)\,dx}$$

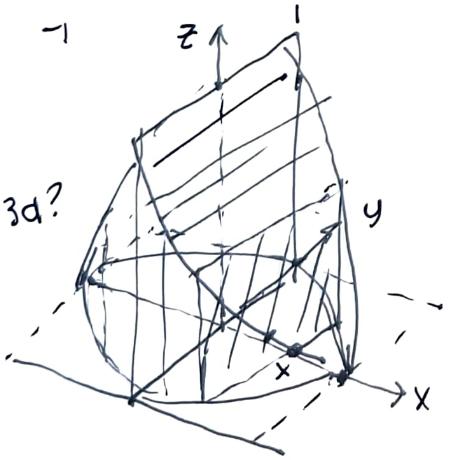
even symmetry

$$= 8\left(x - \frac{2x^3}{3} + \frac{x^5}{5}\right)\Big|_0^1 = 8\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{8(15 - 10 + 3)}{15}$$

$$= \boxed{\frac{64}{15}}$$



3d?



not required

See Maple answer key for pretty plot.

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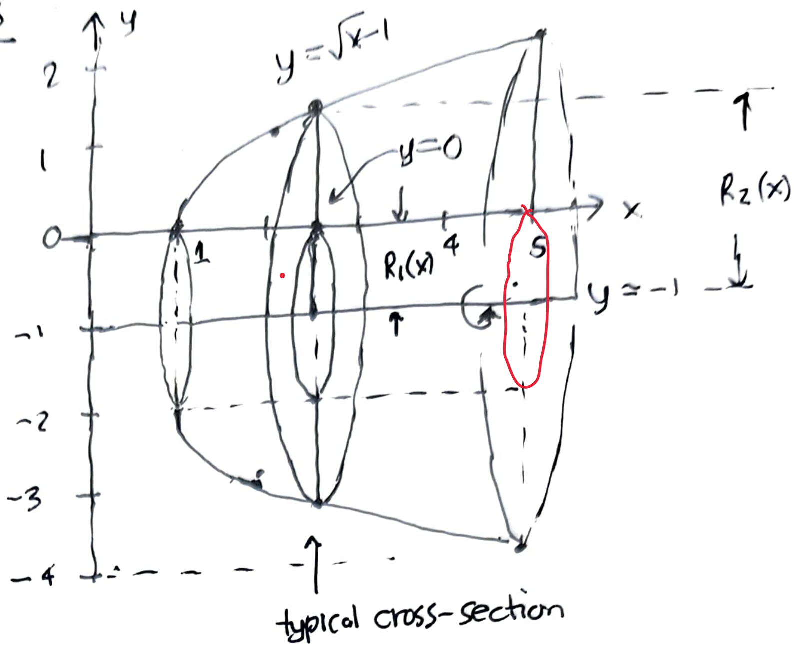

(2) $y=\sqrt{x-1}$, $x \geq 1$

$y=0$ (x-axis)

$x=5$

$R_2(x) = \sqrt{x-1}-(-1)$
$= \sqrt{x-1}+1$

$R_1(x) = 0-(-1) = 1$

$A(x) = \pi(R_2(x)^2 - R_1(x)^2)$
$= \pi\left((\sqrt{x-1}+1)^2 - 1^2\right)$
$= \pi\left((x-1) + 2\sqrt{x-1} + \cancel{1-1}_0\right)$
$= \pi\left(x-1+2\sqrt{x-1}\right)$

$V = \int_1^5 A(x)\,dx = \int_1^5 \pi\left(x-1+2\underbrace{\sqrt{x-1}}\right)dx$

side calculation

$\int 2(\underbrace{x-1}_{u})^{1/2} \underbrace{dx}_{du}$
$= 2\dfrac{u^{3/2}}{3/2} + C$
$= \frac{4}{3}u^{3/2} + C$
$= \frac{4}{3}(x-1)^{3/2} + C$

$= \pi\left(\frac{x^2}{2} - x + \frac{4}{3}(x-1)^{3/2}\right)\Big|_1^5$

$= \pi\left(\frac{25}{2} - 5 + \frac{4}{3}2^{3/2} - \left(\frac{1}{2} - 1 + 0\right)\right)$

~~$= \pi\left(\frac{15}{2} + \frac{32}{3}\right) = \pi\frac{45+32}{6} = \frac{72\pi}{6}$~~ sloppy! be careful, bob!

~~$= \pi\left(\frac{25+1}{2} + \frac{4}{3}\cdot 8\right) = \pi\left(13 + \frac{32}{3}\right)$~~ again! (use technology for this evaluation!)

$= \pi\left(\frac{25-10+1}{2} + \frac{4\cdot 8}{3}\right) = \pi\left(8+\frac{32}{3}\right) = \pi\left(\frac{24+32}{3}\right) = \boxed{\frac{56\pi}{3}}$