MAT1505-05 23F Quiz 2   Print Name (Last, First) _____|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). Technology can only be used to check hand calculations and not substitute for them.

1. An internet search finds the claim: "US adult female heights are on average 64.5 inches (5'4 1/2") with a standard deviation of 2.5 inches." The fraction F of such adult females within two standard deviations above the mean value ("average") for a normal distribution is given by the following definite integral

$$F = \int_{64.5}^{69.5} \frac{1}{2.5\sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{x-64.5}{2.5}\right)^2} \, dx$$

a) Use Maple to evaluate this exactly and approximate the result to 3 decimal places.

b) Use the change of variable to the "standard variable" $u = \dfrac{x-64.5}{2.5}$ to transform this integral to its standard form in terms of that new variable (which measures the deviation from the average value in multiples of the standard deviation).

c) Use Maple to evaluate the new integral exactly and approximate it to 3 decimal places.

d) They should agree. If not find your error.

e) What percent of this population has a height within two standard deviations above the mean value of 64.5 inches? Answer in a complete English sentence.

2. a) Evaluate $\displaystyle\int_0^a x\sqrt{a^2 - x^2}\, dx$ (assuming $a > 0$) showing all the steps by an appropriate u-substitution, converting the integral entirely to the new variable in this process before evaluating it.

b) Does Maple or your graphing calculator or tech tool confirm your result? Explain.

▶ **solution**

decimals in Maple trigger decimal responses

① a) $$F = \int_{64.5}^{69.5} \frac{1}{2.5\sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{x-64.5}{2.5}\right)^2} \, dx = 0.4772498681$$

$$\approx \boxed{0.477}$$

b) $u = \dfrac{x-64.5}{2.5}, \quad du = \dfrac{dx}{2.5}, \quad dx = 2.5\,du$

$$F = \int_{x=64.5}^{x=69.5} \frac{1}{\sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{x-64.5}{2.5}\right)^2} \underbrace{\frac{dx}{2.5}}_{=du}$$

$x \approx 69.5 \rightarrow u = \dfrac{69.5-64.5}{2.5} = \dfrac{5.0}{2.5} = 2$

$x = 64.5 \rightarrow u = \dfrac{64.5-64.5}{2.5} = 0$

$$= \int_0^2 \frac{1}{\sqrt{2\pi}} e^{-u^2}\, du = \boxed{\frac{\text{erf}(\sqrt{2})}{2}} \approx 0.4772498681 \approx \boxed{0.477}$$

↑ no decimals — exact result

d) ☺ They agree!

e) About 47.7 or 48 percent of the US adult female population has a height within 2 standard deviations above the mean value of 64.5 in.

② a) $\int_0^a x\sqrt{a^2-x^2}\,dx = \int_{x=0}^{x=a} \underbrace{x\,(a^2-x^2)^{1/2}}\,dx$ $= -\frac{1}{2}du$

$$u = a^2 - x^2 \qquad\qquad x=a \rightarrow u = a^2-a^2 = 0$$
$$du = -2x\,dx \qquad\qquad x=0 \rightarrow u = a^2-0 = a^2$$
$$-\frac{1}{2}du = x\,dx$$

$= \int_{a^2}^{0} u^{1/2}\left(-\frac{1}{2}du\right) = \frac{1}{2}\int_0^{a^2} u^{1/2}\,du = \frac{1}{2}\frac{u^{3/2}}{3/2}\Big|_0^{a^2}$

$= \frac{1}{3}u^{3/2}\Big|_0^{a^2} = \frac{1}{3}(a^2)^{3/2} - 0 = \boxed{\frac{1}{3}a^3}$ (if $a \geq 0$!)

b) Maple gives: $\frac{1}{3}a^3\,\underbrace{\mathrm{csgn}(a)}$

complex sign (a)
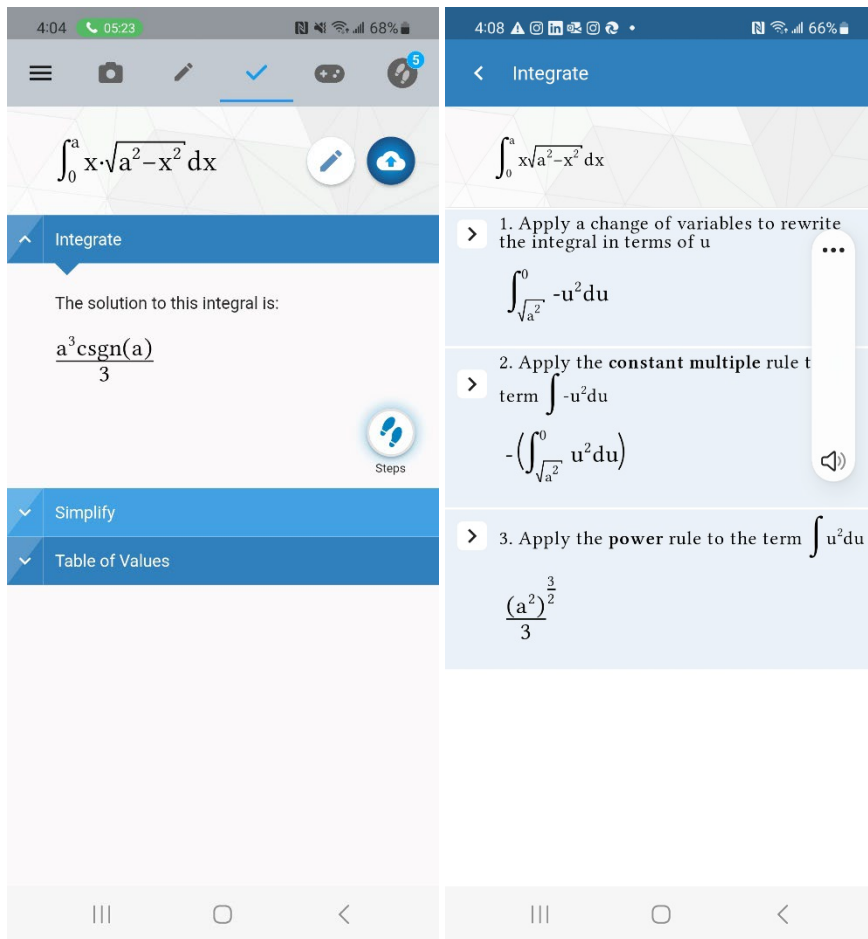$= \mathrm{sign}(a)$ if a is real
$= 1$ if $a > 0$ as assumed

I have no clue what a graphing calculator says. (I never use one.)

Maple calculator (phone app). Scan handwritten formula with the camera icon on the top toolbar & it gives the same result as Maple.

$$\int_0^a x \cdot \sqrt{a^2 - x^2}\, dx$$

Integrate

The solution to this integral is:

$$\frac{a^3 \operatorname{csgn}(a)}{3}$$

Steps

Simplify

Table of Values

< **Integrate**

$$\int_0^a x\sqrt{a^2 - x^2}\, dx$$

> 1. Apply a change of variables to rewrite the integral in terms of u

$$\int_{\sqrt{a^2}}^0 -u^2\, du$$

> 2. Apply the **constant multiple** rule t term $\int -u^2\, du$

$$-\left(\int_{\sqrt{a^2}}^0 u^2\, du\right)$$

> 3. Apply the **power** rule to the term $\int u^2\, du$

$$\frac{(a^2)^{\frac{3}{2}}}{3}$$

Maple Calculator, once you scan the handwritten integral into the app. The steps button produces the right screenshot.