MAT1505 23F Quiz 1 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC).

1. Evaluate the following integrals, showing your simplication steps and antiderivatives using correct notation, starting your continuing equality $A = B = C = \ldots$ by stating the definite integral as the starting left hand side.

a) $\int_1^3 \left(\frac{1}{z^2} + \frac{1}{z^3} \right) dz$, b) $\int_1^3 \frac{y^3 - 2y^2 - y}{y^2}\, dy$

2. On what interval is the following function concave down? Show all your work and explain your reasoning. Sketch the graph of the second derivative roughly on the interval $[-10, 10]$ from a technology screen, identifying axes and axis tickmarks (or print to paper and attach).

$$F(x) = \int_0^x \frac{t^2}{t^2 + t + 2}\, dt .$$

▶ solution

① a) $\int_1^3 \frac{1}{z^2} + \frac{1}{z^3}\, dz = \int_1^3 z^{-2} + z^{-3}\, dz = \frac{z^{-1}}{-1} + \frac{z^{-2}}{-2} \Big|_1^3 = -\frac{1}{z} - \frac{1}{2z^2} \Big|_1^3$

$= -\frac{1}{3} - \frac{1}{2\cdot 9} - \left(-1 - \frac{1}{2}\right) = \frac{3}{2} - \frac{1}{3} - \frac{1}{6\cdot 3} = \frac{7}{6} - \frac{1}{3(6)} = \frac{1}{6}\left(7 - \frac{1}{3}\right) = \frac{1}{6}\cdot\frac{20}{3} = \boxed{\frac{10}{9}}$

$=$ or use tech $\boxed{\frac{10}{9}}$

b) $\int_1^3 \frac{y^3 - 2y^2 - y}{y^2}\, dy = \int_1^3 y - 2 - \frac{1}{y}\, dy = \frac{y^2}{2} - 2y - \ln y \Big|_1^3$

$= \frac{9}{2} - 6 - \ln 3 - \left(\frac{1}{2} - 2 - \ln 1\right) = \frac{9}{2} - 4 - \ln 3 = \boxed{-\ln 3}$

$=$ or use tech $\boxed{-\ln 3}$

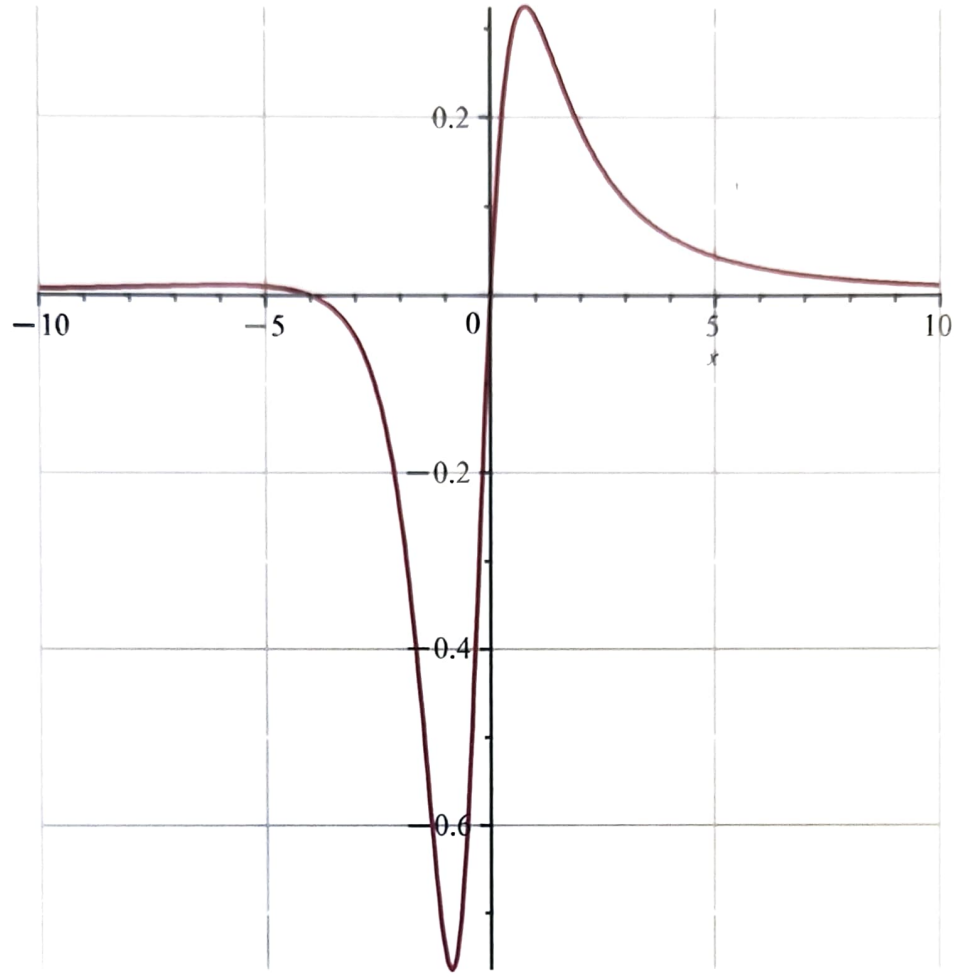
② $F(x) = \int_0^x \frac{t^2}{t^2+t+2}\, dt \rightarrow F''(x) < 0$ for concave down graph

$F'(x) = \frac{x^2}{x^2+x+2}$ (↑ FTC), $F''(x) = \frac{(x^2+x+2)(2x) - x^2(2x+1)}{(x^2+x+2)^2} = \frac{2x^3 + 2x^2 + 4x - 2x^3 - x^2}{(x^2+x+2)^2}$

$= \frac{x^2 + 4x}{(x^2+x+2)^2} = \frac{x(x+4)}{(x^2+x+2)^2} = 0 \rightarrow x = 0, -4$

$F''(-1) = -\frac{(-1+4)}{4^2} = -\frac{3}{4^2} < 0$ so $\boxed{-4 < x < 0}$: $F''(x) < 0$

> $plot\left(\frac{x\,(x+4)}{(x^2+x+2)^2}, x=-10..10\right)$



rough hand drawn sketch