MAT1505-02/03 23F Final Exam Print Name (Last, First) _____ , _____|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if useful). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.** Numerical values can be evaluated with technology.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. For the parametrized curve $x = \frac{1}{2}\left(t^2 + 3\right), y = 5 - \frac{1}{3}t^3, 0 \le t \le 3$:

a) Derive the equation of the tangent line at $t = 1$.

b) Simplify the integral formula for the arclength of this curve and evaluate it exactly by hand (factor and u-sub).

2. For the parametrized curve $x = t^3 - 3t, y = t^3 - 3t^2, -1.5 \le t \le 2.5$:

a) Find the coordinates of the points on the curve where the tangent lines are horizontal or vertical, justifying your values.

b) Make a rough sketch of the plot of this parametrized curve segment indicating these points with their coordinates and parameter values, and indicate the curve's direction with arrows along the curve.

3. Consider the limacon $r = 1 - 2\sin(\theta)$.

a) Make a rough sketch of this polar curve from your PlotBuilder plot or by simply reasoning it out.

b) What continous angular range of θ corresponds to the inner loop of this curve? Explain.

c) Set up an integral representing the area of this inner loop and simplify it.

d) Evaluate it exactly using technology.

e) Approximate it to 3 decimal places.

f) Set up an integral representing the arclength of the entire polar curve and simplify it [multiply out and use a trig identity].

g) Evaluate it exactly using technology and approximate it to 3 decimal places.

4) Consider the 4 petal polar curve $r = 2\cos(2\theta)$.

a) Make a rough sketch of this polar curve from your PlotBuilder plot or by simply reasoning it out.

b) What continuous angular range $a \le \theta \le b$ corresponds to the right-most loop around the positive x-axis? Explain.

c) Set up an integral representing the area A of this loop and simplify it.

d) Evaluate it exactly using technology and approximate it to 3 decimal places.

e) How does this value compare to the area of the circumscribed circle $r = 2\cos(\theta)$ with diameter 2 (fraction or percentage)? If you plot them together with PlotBuilder to compare visually, does this seem reasonable?