MAT1505-02/03 23F Test 3   Print Name (Last, First) ________________ , ______________|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). INDICATE where technology is used and what type (Maple, GC). **Technology can only be used to check hand calculations and not substitute for them, unless specifically stated.** Numerical values can be evaluated with technology.

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet in on top of your answer sheets as a cover page, with the first test page facing up:
"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

1. Consider the infinite series $\sum_{n=0}^{\infty}\left(\frac{3\,(x-6)^n}{(2n+1)\,2^{2n+1}}\right)$.

a) Determine the radius of convergence R.

b) Determine the complete interval of convergence, supporting your conclusions about endpoint convergence, using words to explain if helpful.

2. Use the lowest nonzero terms in the Taylor series representation of the numerator of the following limit (starting from the binomial series formula) in order to evaluate it (show one more term than necessary in evaluating the limit, using " $+\ldots$" notation).

$$\lim_{x\to 0}\left(\frac{(1+2x^2)^{\frac{3}{2}}-1-3x^2}{x^4}\right).$$

3. Consider the definite integral $\int_0^{0.4} x^2\,e^{-\frac{x^2}{2}}\,dx$ and use a power series representation of the integrand to approximate it to an error less then 10^{-6} by following the steps:

a) Use the Taylor series representation of e^x to express $f(x)=x^2\,e^{-\frac{x^2}{2}}$ as a power series, stating the summation formula involving the simplified nth term in the series.

b) Give the power series which results from term by term integration $\int x^2\,e^{-\frac{x^2}{2}}\,dx$ setting the constant of integration to zero to get the antiderivative $F(x)$ which vanishes at $x=0$.

c) Write out explicitly the first four nonzero terms in this indefinite integral series.

(see reverse side)

d) Use it to evaluate the truncated definite integral power series for $\int_0^{0.4} x^2 e^{-\frac{x^2}{2}}\, dx$ consisting of the first four terms without simplifying the powers of 0.4, so that you can evaluate these terms before combining any number of them below.
e) Evaluate the numerical value of each of these 4 terms.
f) Using the fact that this is an alternating series, how many terms should be needed to get the desired accuracy?
g) Evaluate the partial sum for this number of terms and round the result to 5 decimal places.
It should agree to 5 decimal places with Maple's value for this definite integral. Does it? Please respond yes or no.

4. The electric potential due to point charges q and $-q$ located at $r = x$ and $r = -x$ on the r-axis is given by the formula

$$V = \frac{q}{r - x} - \frac{q}{r + x} .$$

a) Using the geometric series trick to find the two lowest (nonzero) terms in the power series representation of V appropriate to approximate its value for points on the axis very far from those charges for which $|r|$ is much greater than $|x|$. [Hint: Start by factoring out r from the denominator.]
b) For what values of $|x|$ is the second term less than one percent of the first term?

▶ solution [don't write on this page please]