MAT 1505-02/03 23F test 2

1a) $\displaystyle\int_2^4 \frac{x}{\sqrt{x-2}}\,dx = \lim_{t\to 2+} \int_t^4 \frac{x}{\sqrt{x-2}}\,dx$ since the integrand $\to \infty$ as $x \to 2+$.

b) $\displaystyle\int \frac{x}{\sqrt{x-2}}\,dx = \int \frac{(u+2)}{\sqrt{u}}\,du = \int u^{1/2} + 2u^{-1/2}\,du$

$[u = x-2 \to x = u+2,\ dx = du]$

$\displaystyle = \frac{u^{3/2}}{3/2} + 2\frac{u^{1/2}}{1/2} + C = \boxed{\tfrac{2}{3}(x-2)^{3/2} + 4(x-2)^{1/2} + C}$

$\displaystyle = \underbrace{\left(4 + \tfrac{2}{3}(x-2)\right)}_{\frac{12+2x-4}{3} = \frac{8+2x}{3} = \frac{2}{3}(x+4)}(x-2)^{1/2} + C = \boxed{\tfrac{2}{3}(x+4)\sqrt{x-2} + C}$

alternative

c) $\displaystyle\int_2^4 \frac{x}{\sqrt{x-2}}\,dx = \lim_{t\to 2^-} \left(\tfrac{2}{3}(x-2)^{3/2} + 4(x-2)^{1/2}\right)\Big|_t^4$

$= \tfrac{2}{3}(2)^{3/2} + 4\cdot 2^{1/2} - 0 = \left(\tfrac{2}{3}\cdot 2 + 4\right)2^{1/2} = \boxed{\tfrac{16}{3}\sqrt{2}}$

2 a) $\displaystyle y = \frac{(x^2+2)^{3/2}}{3} \qquad \frac{dy}{dx} = \frac{3}{2}\,\frac{(x^2+2)^{1/2}(2x+0)}{3} = x(x^2+2)^{1/2}$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^2(x^2+2) = 1 + 2x^2 + x^4 = (1+x^2)^2$

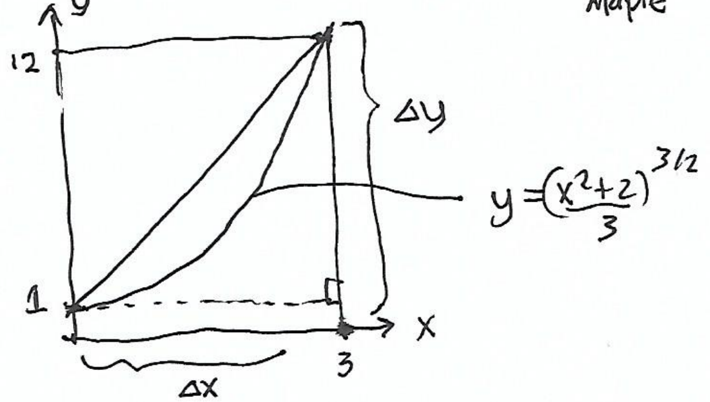
$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 1 + x^2$

$\displaystyle s = \int_0^3 (1+x^2)\,dx = x + \frac{x^3}{3}\Big|_0^3 = 3 + 3^2 = \boxed{12}$

b) $\displaystyle y(0) = \frac{2^{3/2}}{3}, \quad y(3) = \frac{(9+2)^{3/2}}{3} = \frac{11^{3/2}}{3}$

$\left.\begin{array}{l}\Delta y = y(3) - y(0) = \dfrac{11^{3/2} - 2^{3/2}}{3} \\ \Delta x = 3 - 0 = 3\end{array}\right\}\ L = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{9 + \dfrac{(11^{3/2} - 2^{3/2})^2}{9}}$

$\underset{\text{Maple}}{\approx} \boxed{11.612}$ a bit less than 12 as expected



not required:
Note $y(0) \approx 0.94$
$y(3) \approx 12.16$
explains tickmarks

MAT 1505-02/03 23F Test 2 (2)

③ a) $\int_0^\infty 3x^2 e^{-x^3}dx = \lim_{t\to\infty}\int_0^t 3x^2 e^{-x^3}dx$

$$\int 3x^2 e^{-x^3}dx = \int e^u(-du) = -e^u + C = -e^{-x^3}+C$$

$$\left[\begin{array}{l} u=-x^3 \\ du=-3x^2dx \end{array}\right]$$

$$\int_0^\infty 3x^2 e^{-x^3}dx = \lim_{t\to\infty} -e^{-x^3}\Big|_0^t = \lim_{t\to\infty}\left(-e^{-t^3}+e^0\right)$$

$$= \lim_{t\to\infty}\left(1-\underbrace{e^{-t^3}}_{\to 0}\right) = 1 \quad ✓$$

b) $p(x) = 3x^2 e^{-x^3}$

$p'(x) = 3(2x)e^{-x^3} + 3x^2e^{-x^3}(-3x^2)$

$= 3xe^{-x^3}(2-3x^3) = 0 \rightarrow x=0,\ x^3=\frac{2}{3},\ x=\boxed{\left(\frac{2}{3}\right)^{1/3}=\bar{X}}$ ≈ 0.8736

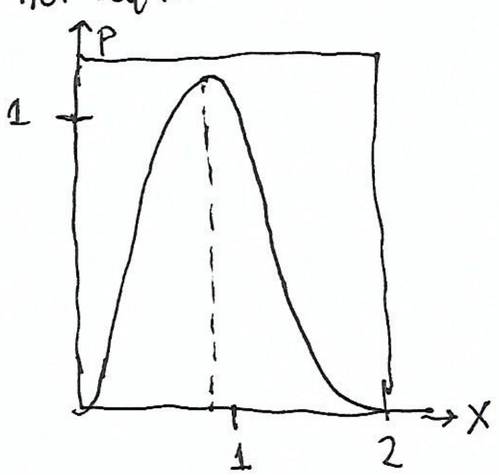
$$\int_0^{\bar{X}} 3x^2e^{-x^3}dx = -e^{-x^3}\Big|_0^{\bar{X}} = -e^{-\bar{X}^3}+1 = 1-e^{-2/3}$$ ≈ 0.87

≈ 0.48658

$\approx \boxed{0.4866}$

$\rightarrow \boxed{48.7\%}$

not required:



optional c): median m:

$$\frac{1}{2} = \int_0^m 3x^2e^{-x^3}dx = -e^{-x^3}\Big|_0^m$$

$$= -e^{-m^3}+e^0 = 1-e^{-m^3}$$

$e^{-m^3} = 1-\frac{1}{2} = \frac{1}{2}$

$e^{m^3} = 2$

$m^3 = \ln 2 \qquad m = (\ln 2)^{1/3} \approx 0.884997$

≈ 0.8850

a bit larger than where the peak occurs