MAT1505-02/03 23F Test 1B   Print Name (Last, First) _____ , _____|____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of each problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them. Any results easily checked with technology which are wrong will result in points lost. No collaboration or help from the Math Center is permitted on this take home exam.

▼ pledge

1. Consider the function $f(x) = x^3 \sqrt{4 - x^2}$, $0 \le x \le 2$.

a) Use a change of variable to evaluate $\int f(x)\, dx$ and simplify it as much as possible.

b) Use the particular antiderivative you found in part a) to evaluate the average value f_{avg} of this function on the given interval.

c) Find all the numerical values of x where the function equals its average value on this interval to 4 decimal places.

[The exact value requires solving a 4th degree equation which we cannot do and Maple's exact formula is useless.]

d) Print out a technology plot of the function on this interval together with its average value line (as a constant function).

Annotate it by hand with the numerical value of the average, and mark the points on the x-axis where the function equals its average value annotated by the 2 decimal place values.

2. Consider the region R of the plane between the two graphs $y = x^2$, $y = 1 - x^2$ and rotate it about the axis $y = 2$ to create a volume of revolution.

a) Find the intersection points of the two graphs.

b) Make a diagram showing R and the rotation axis shading in this region and indicating a single vertical linear cross-section at a typical value of x roughly somewhere in the interior away from the ends of this region, and clearly mark the two relevant radii on it using braces or a double arrow or whatever to mark off the endpoints for each radius like bob has done in all of his examples.

c) From your diagram state explicitly these two radius functions of x (make sure both are nonnegative).

d) Evaluate the cross-sectional area of the volume of revolution in terms of them and simplify it to a polynomial.

e) Evaluate the volume of this solid exactly and then to 4 decimal places.

[turn over!]

3. a) Evaluate $\displaystyle\int_0^{\frac{T}{2}} At\ \sin\left(\frac{2\pi\ t}{T}\right)\,dt$ exactly with technology.

b) Change the variable in this definite integral by the rescaling transformation $w = \dfrac{2\pi\ t}{T}$ to re-express it entirely in terms of the new variable without evaluating it yet.

c) Now evaluate this new integral by hand.

4. Given the velocity profile $v(r) = \dfrac{P\,R^2}{4\,\eta\,\ell}\cos\left(\dfrac{\pi\,r}{2\,R}\right)$ for $0 \le r \le R$ for blood flow in a long cylindrical tube:

a) Make a rough sketch of this function on this interval from your knowledge of the cosine function, labeling the vertical axis by its maximum value v_{max}. [Hint: what are the endpoint values?]

b) What is the value of v_{max} ?

c) Evaluate the average velocity for this profile by hand as a function of these 4 parameters, and then as a multiple of v_{max} exactly, Evaluate the coefficient of this multiple to 2 decimal places. What percent of the maximum velocity does this numerical fraction represent?

▶ **solution**