MAT2705-04/05 22S Quiz 8 Print Name (Last, First) ____________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $9\,y''+6\,y'+82\,y=0,\ \ y(0)=3, y'(0)=11$ [prime is d/dt , i.e., the independent variable is the time t not x]

a) Put the DE into standard linear form $y''+k_0 y'+\omega_0^2\,y=0$ first. Then identify the values of the damping constant $k_0=1/\tau_0$, the natural frequency ω_0 , and the numerical value of the quality factor $Q=\omega_0\,\tau_0$.

b) Find the general solution by hand, showing all steps. [Find the roots with Maple!]

c) Find the solution satisfying the initial conditions, showing all steps. [Does your solution of the IVP agree with Maple?]

d) Re-express the sinusoidal factor of this solution exactly in phase-shifted cosine form (evaluating the amplitude and phase shift first exactly), obtaining the two envelope functions of this decaying oscillation solution. State the two envelope functions. State the phase shift in degrees to the nearest degree. What fraction of a cycle (2π) is the phase shift approximately?

e) Print and scan your Maple sketch or make a rough sketch of the plot of your solution and its two envelope functions in a viewing window of width 5 times the characteristic time of the solution exponential factor. How many oscillations approximately can you count in this window?

f) Use calculus by hand to determine ***exactly*** the values of t and y of the first maximum of the solution function $y(t)$ for $t\ge 0$ and their 2 decimal place approximations. [Use Maple to get the exact formulas!]

► solution

a) $9y''+6y'+82y=0$

$\boxed{y''+\frac{2}{3}y'+\frac{82}{9}y=0} \rightarrow \boxed{k_0=\frac{2}{3},\ \tau_0=\frac{3}{2}=1.5}$

$\omega_0^2=\frac{82}{9}$

$\omega_0=\frac{\sqrt{82}}{3}\approx 3.018$

$\boxed{Q=\omega_0\tau_0=\frac{\sqrt{82}}{2}}$

$\boxed{\approx 4.53} >> \frac{1}{2}$

weakly damped

$y=e^{rx}$

b) $r^2+\frac{2}{3}r+\frac{82}{9}=0$

or

$9r^2+6r+82=0$

$r=\frac{-6\pm\sqrt{6^2-4\cdot 3^2\cdot 82}}{2\cdot 9}$

$=\frac{-6\pm 6\sqrt{1-82}}{6\cdot 3}=-\frac{1}{3}\pm 3i$

(or use Maple) $k=\frac{1}{3},\ \omega=3\rightarrow\tau=3\rightarrow 5\tau=15$

$e^{rt}=e^{(-\frac{1}{3}\pm 3i)t}=e^{-t/3}(\cos 3t\pm i\sin 3t)$

↳ real basis: $e^{-t/3}\cos 3t,\ e^{-3t}\sin 3t$

gen soln:

$\boxed{y=e^{-t/3}(c_1\cos 3t+c_2\sin 3t)}$

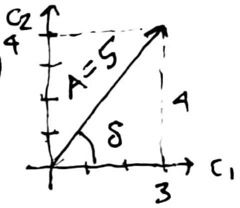
c) $y'=-\frac{1}{3}e^{-t/3}(c_1\cos 3t+c_2\sin 3t)$
$\quad+e^{-t/3}(-3c_1\sin 3t+3c_2\cos 3t)$

$y(0)=c_1=3$

$y'(0)=-c_1/3+3c_2=11\rightarrow c_2=\frac{1}{3}(11+\frac{1}{3}(3))=4$

$\boxed{y=e^{-t/3}(3\cos 3t+4\sin 3t)}$

d)



$A=\sqrt{3^2+4^2}=5$

$\tan\delta=4/3$

$\delta=\arctan 4/3$

$\boxed{\approx 53.1°}$

$\boxed{\approx 0.15 \text{ cycles}}$

$\boxed{y=5e^{-t/3}\cos(3t-\arctan 4/3)}$

e) envelopes: $y=\pm 5e^{-t/3}$

$t=0..5\tau=0..15$

Period $T=2\pi/3\approx 2.09$

$\frac{5\tau}{T}\approx 7.16$ should see about $\boxed{7 \text{ cycles}}$

see page 2 for plot

f) $y'=-\frac{1}{3}e^{-t}(3\cos(3t)+4\sin 3t)$
$\quad+e^{-t/3}(-9\sin 3t+12\cos 3t)$

$=e^{-t/3}(11\cos 3t-\frac{31}{3}\sin 3t)=0$

$\tan 3t=\frac{33}{31}$, $\boxed{t=\frac{1}{3}\arctan\frac{33}{31}\approx 0.27}$

$\boxed{y\approx 4.54}$ (Maple)

This confirms what we see in the plot

>

> $plot\left(\left[4\,e^{-\frac{t}{3}}\,\sin(3\,t)+3\,e^{-\frac{t}{3}}\,\cos(3\,t),5e^{-\frac{t}{3}},-5e^{-\frac{t}{3}}\right],t=0..15\right)$