MAT2705-04/05 22S Quiz 7 Print Name (Last, First) ____________________

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use equal signs and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Maple can only be used to check not justify responses.

Consider the IVP: $2y''+5y'+2y=0,\ y(0)=1,\ y'(0)=4$ [independent variable: x].

a) Find a basis $\{y_1, y_2\}$ of the solution space using the exponential technique. [Order doesn't matter!]

b) Use that basis to express the general solution $y(x)$ of the DE.

c) Using 2x2 matrix methods involving the inverse matrix, find the solution which satisfies the initial conditions, showing all work (report the value of the inverse matrix you use).

d) What are the two characteristic lengths $0 < \tau_1 < \tau_2$ for the two exponentials in this problem? [Let τ_2 be the larger one.]

e) Use technology to plot your result for $x = 0..5\,\tau_2$ and an appropriate vertical window and make a rough sketch of what you see, labeling the axes with variable names and key tickmarks on your sketch. (Clicking on the gridlines icon for a Maple plot helps you make your hand sketch more accurate.)

f) Use calculus to determine ***exactly*** (rules of exponents and logs! simplify your result for y to a single simple term) the x and y values of the point of obvious global maximum you see on the graph and then their approximate values to 2 decimal places. Locate on your sketch and label the maximum point by its 2 decimal plaee coordinates . Do the numbers you found agree with what your eyes see in the technology plot?

▶ solution

a) $2y''+5y'+2y=0$

$y=e^{rx} \rightsquigarrow (2r^2+5r+2)e^{rx}=0$

$2r^2+5r+2=0$

$r=\frac{-5\pm\sqrt{25-4(2)(2)}}{2(2)} = \frac{-5\pm\sqrt{9}}{4} = \frac{-5\pm 3}{4}$

$= -\frac{1}{2}, -2$

$y=e^{rx} = \underbrace{e^{-\frac{x}{2}}}_{y_1}, \underbrace{e^{-2x}}_{y_2}$ $\boxed{\{e^{-\frac{x}{2}}, e^{-2x}\} \text{ is a basis}}$

b) gen soln: $y=c_1e^{-\frac{x}{2}}+c_2e^{-2x}$

c) $y'=-\frac{1}{2}c_1e^{-\frac{x}{2}}-2c_2e^{-2x}$

$y(0)=c_1+c_2=1$

$y'(0)=-\frac{1}{2}c_1-2c_2=4$

$\begin{bmatrix}1&1\\-\frac{1}{2}&-2\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}1\\4\end{bmatrix}$

inverse matrix $-\frac{2}{3}$

$\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}1&1\\-\frac{1}{2}&-2\end{bmatrix}^{-1}\begin{bmatrix}1\\4\end{bmatrix}=\frac{1}{-2+\frac{1}{2}}\begin{bmatrix}-2&-1\\\frac{1}{2}&1\end{bmatrix}\begin{bmatrix}1\\4\end{bmatrix}$

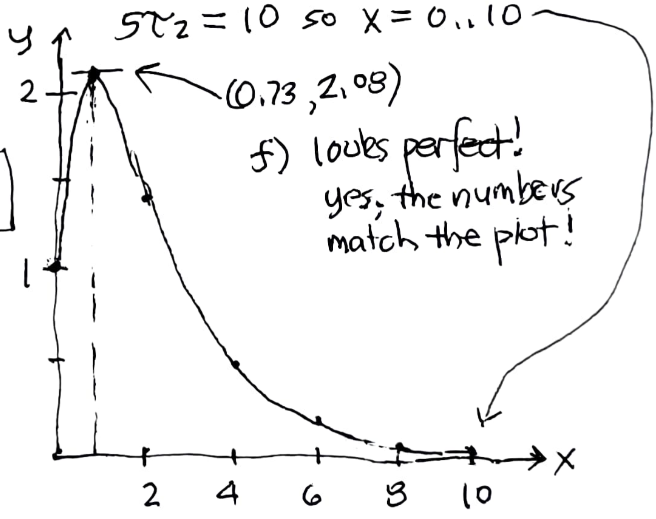
$=-\frac{2}{3}\begin{bmatrix}-2-4\\\frac{1}{2}+4\end{bmatrix}=\frac{2}{3}\begin{bmatrix}6\\-9/2\end{bmatrix}=\begin{bmatrix}4\\-3\end{bmatrix}$

$\boxed{y= 4e^{-x/2}-3e^{-2x}}$

d) $e^{-\frac{x}{2}} \to \tau = 2 = \tau_2$

$e^{-2x} \to \tau = \frac{1}{2} = \tau_1$

$5\tau_2 = 10$ so $x=0..10$



f) looks perfect! yes, the numbers match the plot!

f) $0=y'=4(-\frac{1}{2})e^{-x/2}-3(-2)e^{-2x}$

$=-2e^{-x/2}+6e^{-2x}\ ]\ e^{2x}/2$

$-e^{(2-\frac{1}{2})x}+3=0$

$e^{3/2\,x}=3 \to x=\boxed{\frac{2}{3}\ln 3 \approx 0.73}$

$y=4e^{-\frac{1}{2}(\frac{2}{3}\ln 3)}-3e^{-2(\frac{2}{3}\ln 3)}$

$=4(e^{\ln 3})^{-1/3}-3(e^{\ln 3})^{-4/3}$

$=\frac{4}{3^{1/3}}-\frac{3}{3^{4/3}}=\frac{1}{3^{1/3}}(4-1)=\boxed{3^{2/3}\approx 2.08}$

$\frac{3}{3\cdot 3^{1/3}}=\frac{1}{3^{1/3}}$

or simplify with Maple