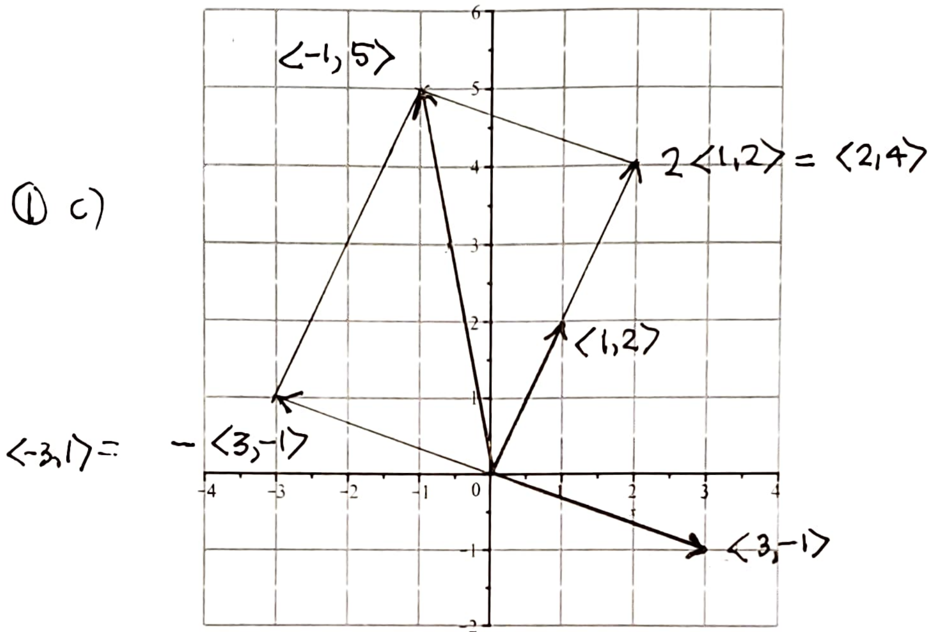
MAT2705-04/05 22S Quiz 6 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

1. a) Express the vector $\langle -1, 5\rangle$ as a linear combination of the vectors $\{\langle 3,-1\rangle, \langle 1, 2\rangle\}$.
b) Show that this linear combination evaluates to the target vector.
c) Assuming $\langle -1, 5\rangle = y_1\langle 3,-1\rangle + y_2\langle 1, 2\rangle$, make a diagram on the grid ~~on page 2~~ showing the three vectors $\langle -1, 5\rangle, y_1\langle 3,-1\rangle, y_2\langle 1, 2\rangle$ and the parallelogram formed by the last two vectors. [Label these three vectors by their components.]



2. a) Express $\langle 11,-2,-1\rangle$ as a linear combination of the vectors $\{\langle 3, 5, 2\rangle, \langle 1, 8, 5\rangle, \langle -3, 2, 0\rangle\}$ by writing down and solving the corresponding matrix equation, using the inverse coefficient matrix.
b) Check by evaluating this linear combination.

3. a) Can $\langle -9,-4, 3\rangle$, be expressed as a linear combination of $\langle 1, 2, 1\rangle, \langle -3, 1, 3\rangle$? If so, do it. If not, show why not.
b) Can $\langle -9,-4, 4\rangle$, be expressed as a linear combination of $\langle 1, 2, 1\rangle, \langle -3, 1, 3\rangle$? If so, so it. If not, show why not.

▶ solution

① a) $y_1\begin{bmatrix}3\\-1\end{bmatrix}+y_2\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}-1\\5\end{bmatrix} \longrightarrow \begin{bmatrix}3&1\\-1&2\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}-1\\5\end{bmatrix} \longrightarrow \begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}3&1\\-1&2\end{bmatrix}^{-1}\begin{bmatrix}-1\\5\end{bmatrix}$

$= \frac{1}{6+1}\begin{bmatrix}2&-1\\+1&3\end{bmatrix}\begin{bmatrix}-1\\5\end{bmatrix} = \frac{1}{7}\begin{bmatrix}-2-5\\-1+15\end{bmatrix} = \begin{bmatrix}-1\\2\end{bmatrix}$

so $\boxed{\begin{bmatrix}-1\\5\end{bmatrix} = -1\begin{bmatrix}3\\-1\end{bmatrix}+2\begin{bmatrix}1\\2\end{bmatrix}}$

(b) $= \begin{bmatrix}-3\\1\end{bmatrix}+\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}-3+2\\1+4\end{bmatrix} = \begin{bmatrix}-1\\5\end{bmatrix}$ ✓

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② a) $y_1\begin{bmatrix}3\\5\\2\end{bmatrix}+y_2\begin{bmatrix}1\\8\\5\end{bmatrix}+y_3\begin{bmatrix}-3\\2\\0\end{bmatrix} = \boxed{\begin{bmatrix}3&1&-3\\5&8&2\\2&5&0\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}11\\-2\\-1\end{bmatrix}}$

b) $\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}3&1&-3\\5&8&2\\2&5&0\end{bmatrix}^{-1}\begin{bmatrix}11\\-2\\-1\end{bmatrix}=\frac{1}{53}\begin{bmatrix}10&15&-26\\-4&-6&21\\-9&13&-19\end{bmatrix}\begin{bmatrix}11\\-2\\1\end{bmatrix} \underset{\text{Maple}}{=} \begin{bmatrix}2\\-1\\-2\end{bmatrix}$

so $\boxed{\begin{bmatrix}11\\-2\\1\end{bmatrix}=2\begin{bmatrix}3\\5\\2\end{bmatrix}-\begin{bmatrix}1\\8\\5\end{bmatrix}-2\begin{bmatrix}-3\\2\\0\end{bmatrix}} \underset{(c)}{=} \begin{bmatrix}6-+6\\10-8-4\\4-5+0\end{bmatrix}=\begin{bmatrix}\\-2\\-1\end{bmatrix}$ ✓

③ a) $x_1\begin{bmatrix}1\\2\\1\end{bmatrix}+x_2\begin{bmatrix}-3\\1\\3\end{bmatrix}=\begin{bmatrix}-9\\-4\\3\end{bmatrix} \to \begin{bmatrix}1&-3\\2&1\\1&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-9\\-4\\3\end{bmatrix}$

$\to \begin{bmatrix}1&-3&-9\\2&1&-4\\1&3&3\end{bmatrix} \xrightarrow[\text{Maple}]{\text{rref}} \begin{bmatrix}1&0&-3\\0&1&2\\0&0&0\end{bmatrix}\begin{matrix}\to x_1=-3\\\to x_2=2\\ \end{matrix}$

L L ..

x_1 x_2

so $\boxed{\begin{bmatrix}-9\\-4\\3\end{bmatrix}=-3\begin{bmatrix}1\\2\\1\end{bmatrix}+2\begin{bmatrix}-3\\1\\3\end{bmatrix}}=\begin{bmatrix}-3-6\\-6+2\\-3+6\end{bmatrix}=\begin{bmatrix}-9\\-4\\3\end{bmatrix}$ ✓

inconsistent system

b) instead $\begin{bmatrix}1&-3&-9\\2&1&-4\\1&3&4\end{bmatrix}\longrightarrow\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{matrix}\to x_1=0\\\to x_2=0\\\to 0=1 \text{ but}\end{matrix}$

x_1 x_2

L L ↑ $0\neq 1$ inconsistent system

no soln