MAT2705-04/05 22S Quiz 6   Print Name (Last, First) ____________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

1. $\begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$

a) Write down the inverse coefficient matrix either from memory or using technology and use it to solve this system, showing the matrix multiplication steps of pairing like entries before adding their products and simplifying.

b) Write down the system of scalar equations equivalent to this matrix equation, and backsubstitute the solution into each equation to verify that your solution does solve this system.

2. Express this linear system in matrix form, evaluate the determinant of the coefficient matrix with technology to show that it is invertible, and use the (technology delivered) inverse matrix to solve the system (state that matrix explicitly):

$x_1 - 3\,x_2 - 3\,x_3 = -3,\ -x_1 + x_2 + 2\,x_3 = 9,\ 2\,x_1 - 3\,x_2 - 3\,x_3 = 0$.

Show the matrix multiplication steps of pairing like entries before adding their products and simplifying.

▶ solution

① a) $A = \begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix}$ $\det(A) = 4(9) - 6(5) = 36 - 30 = 6 \neq 0$ $b = \begin{bmatrix} 6 \\ 6 \end{bmatrix} \to Ax = b$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\downarrow$ $x = A^{-1}b$

$\boxed{A^{-1} = \frac{1}{6}\begin{bmatrix} 9 & -6 \\ -5 & 4 \end{bmatrix}}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 9 & -6 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 9 & -6 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9-6 \\ -5+4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ so $\boxed{x_1 = 3, x_2 = -1}$

or $\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}}$

b) $4x_1 + 6x_2 = 6 \longrightarrow 4(3) + 6(-1) = 12 - 6 = 6$ ✓

$5x_1 + 9x_2 = 9$ $5(3) + 9(-1) = 15 - 9 = 6$ ✓

② $\underbrace{\begin{bmatrix} 1 & -3 & -3 \\ -1 & 1 & 2 \\ 2 & -3 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 \\ 9 \\ 0 \end{bmatrix}}_{b}$

$\boxed{\det(A) = -3} \neq 0$ (Maple)

$\boxed{A^{-1} = \frac{1}{3}\begin{bmatrix} -3 & 0 & 3 \\ -1 & -3 & -1 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1/3 & -1 & -1/3 \\ -1/3 & 1 & 2/3 \end{bmatrix}}$ (either)

↑ Maple

$\boxed{x_1 = 3, x_2 = -8, x_3 = 10}$

or

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3 & 0 & 3 \\ -1 & -3 & -1 \\ -1 & 3 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 9 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3(-3) + 0(9) + 3(0) \\ -1(-3) - 3(9) + (0) \\ -1(-3) + 3(9) + 2(0) \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 3-27 \\ 3+27 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ -24 \\ 30 \end{bmatrix} = \boxed{\begin{bmatrix} 3 \\ -8 \\ 10 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}$