

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, unless specifically requested.

$$2x_1 + 4x_2 - x_3 - 2x_4 + 2x_5 = 6$$

$$x_1 + 3x_2 + 2x_3 - 7x_4 + 3x_5 = 9$$

$$5x_1 + 8x_2 - 7x_3 + 6x_4 + x_5 = 4$$

- Write down the coefficient matrix  $A$ , the RHS matrix  $b$  and the augmented matrix  $C = \langle A \mid b \rangle$  for this linear system of equations.
- Use the LinearSolveTutor to reduce this augmented matrix step by step to its ReducedRowEchelonForm (avoiding fractions as much as possible), recording the intermediate matrices and row operations for each step (as in  $R_1 \leftrightarrow R_2, R_3 \rightarrow R_3 + 2R_1, R_1 \rightarrow \frac{1}{2}R_1$ ) or just copy the output to a blank worksheet, print it on paper, and attach to this quiz to include in your scan.
- Write out the equations that correspond to the reduced matrix. Identify the leading variables (L) and the free variables (F) and solve. State your solution in the scalar form:  $x_1 = \dots, x_2 = \dots$ , etc, then write in column matrix form and identify the coefficient vectors  $u_1, \dots$  of your free variable parameters  $t_1, \dots$  and the constant term vector  $d$  in the form  $x = t_1 u_1 + \dots + d$ .
- Enter the augmented matrix into Maple and by right context panel menu, find the reduced matrix and the solution of the system of equations. Write down exactly what Maple gives you for the column matrix solution and compare with your reduced matrix and solution. They should agree. Do they? How do the parameter symbols correspond?

► **solution**

$$a) \quad A = \begin{bmatrix} 2 & 4 & -1 & -2 & 2 \\ 1 & 3 & 2 & -7 & 3 \\ 5 & 8 & -7 & 6 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 6 \\ 9 \\ 4 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 4 & -1 & -2 & 2 & 6 \\ 1 & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{bmatrix}$$

b) see next 2 pages

> Student[LinearAlgebra][LinearSolveTutor](C);

$$\begin{bmatrix} 2 & 4 & -1 & -2 & 2 & 6 \\ \textcircled{1} & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{bmatrix}$$

Swap rows 1 and 2:

$$\begin{bmatrix} \textcircled{1} & 3 & 2 & -7 & 3 & 9 \\ 2 & 4 & -1 & -2 & 2 & 6 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{bmatrix}$$

Add -2 times row 1 to row 2:

$$\begin{bmatrix} \textcircled{1} & 3 & 2 & -7 & 3 & 9 \\ 0 & -2 & -5 & 12 & -4 & -12 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{bmatrix}$$

Add -5 times row 1 to row 3:

$$\begin{bmatrix} 1 & 3 & 2 & -7 & 3 & 9 \\ 0 & \textcircled{-2} & -5 & 12 & -4 & -12 \\ 0 & -7 & -17 & 41 & -14 & -41 \end{bmatrix}$$

Multiply row 2 by  $-1/2$ :

$$\begin{bmatrix} 1 & 3 & 2 & -7 & 3 & 9 \\ 0 & \textcircled{1} & \frac{5}{2} & -6 & 2 & 6 \\ 0 & -7 & -17 & 41 & -14 & -41 \end{bmatrix}$$

Add -3 times row 2 to row 1:

$$\begin{bmatrix} 1 & 0 & -\frac{11}{2} & 11 & -3 & -9 \\ 0 & 1 & \frac{5}{2} & -6 & 2 & 6 \\ 0 & -7 & -17 & 41 & -14 & -41 \end{bmatrix}$$

Add 7 times row 2 to row 3:

$$\begin{bmatrix} 1 & 0 & -\frac{11}{2} & 11 & -3 & -9 \\ 0 & 1 & \frac{5}{2} & -6 & 2 & 6 \\ 0 & 0 & \textcircled{\frac{1}{2}} & -1 & 0 & 1 \end{bmatrix}$$

Multiply row 3 by 2:

$$\begin{bmatrix} 1 & 0 & -\frac{11}{2} & 11 & -3 & -9 \\ 0 & 1 & \frac{5}{2} & -6 & 2 & 6 \\ 0 & 0 & \textcircled{1} & -2 & 0 & 2 \end{bmatrix}$$

Add  $11/2$  times row 3 to row 1:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & \frac{5}{2} & -6 & 2 & 6 \\ 0 & 0 & 1 & -2 & 0 & 2 \end{bmatrix}$$

Add  $-5/2$  times row 3 to row 2:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 & 2 \end{bmatrix}$$

done

(1)

> `ReducedRowEchelonForm(C)`

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 & 2 \end{bmatrix}$$

$x_1, x_2, x_3$      $x_4, x_5$   
 $L, L, L$          $F, F$

(2)

> `sol := LinearSolve(%)`

$$\text{sol} := \begin{bmatrix} 2 + 3t_5 \\ 1 + t_4 - 2t_5 \\ 2 + 2t_4 \\ -t_4 \\ -t_5 \end{bmatrix}$$

d)

$$\begin{aligned} -t_4 &= t_1 \\ -t_5 &= t_2 \end{aligned}$$

(3)

c) **Leading**:  $\{x_1, x_2, x_3\}$   
**Free**:  $\{x_4, x_5\}$

c)  $x_1$                      $-3x_5 = 2 \rightarrow x_1 = 2 + 3t_2$   
 $x_2$              $-x_4 + 2x_5 = 1 \rightarrow x_2 = 1 + t_1 - 2t_2$   
 $x_3$              $-2x_4 = 2 \rightarrow x_3 = 2 + 2t_1$

$\downarrow$              $\downarrow$              $\left[ \begin{array}{l} x_4 = t_1 \\ x_5 = t_2 \end{array} \right]$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 & +3t_2 \\ 1 + t_1 - 2t_2 \\ 2 + 2t_1 \\ & t_1 \\ & & t_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}}_{u_0} + t_1 \underbrace{\begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}}_{u_1} + t_2 \underbrace{\begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{u_2}$$