MAT2705-04/05 22s Quiz 3 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). **Indicate** where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

1. ....

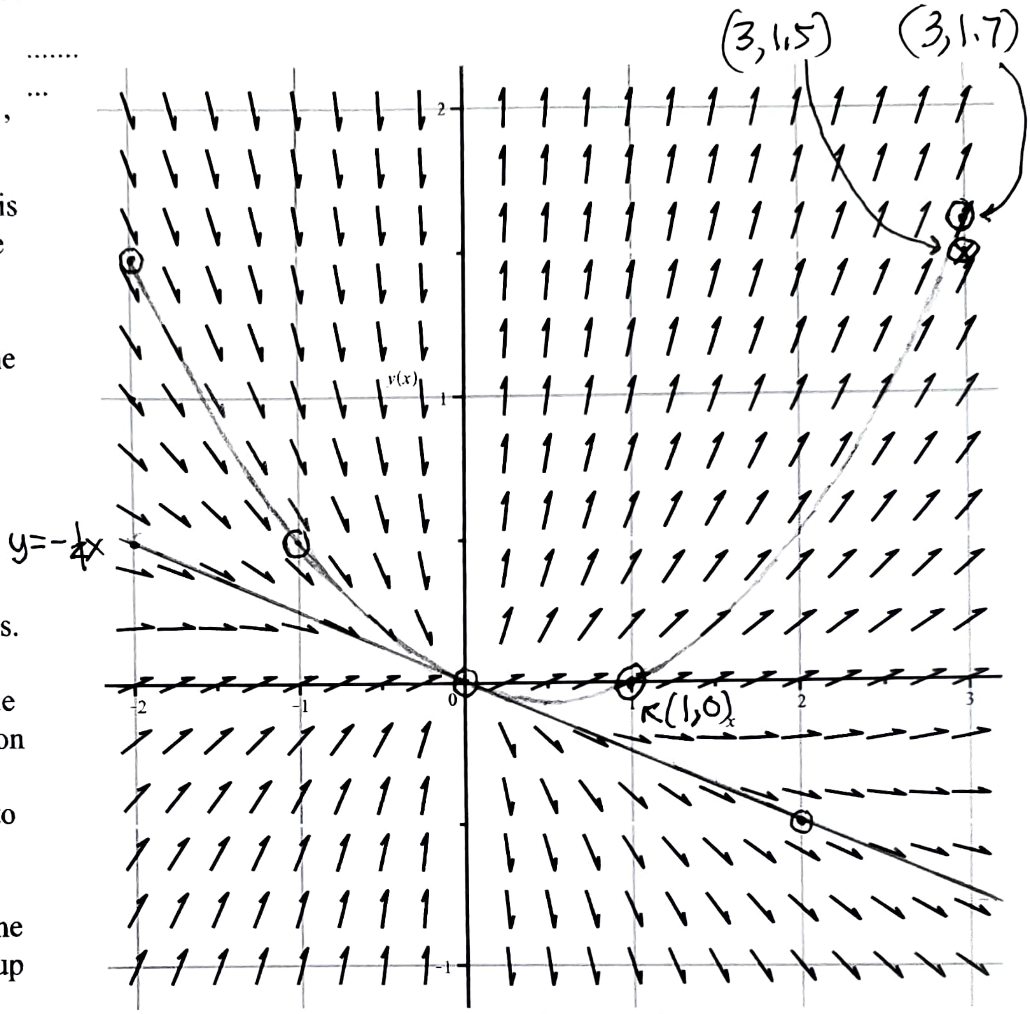
$x\dfrac{dy}{dx} - 2y - \dfrac{x}{4} = 0$, $y(1) = 0$, ...
$x \geq 0$.

a) Hand draw in the solution of this differential equation satisfying the initial condition on the associated direction field to the right for $0 \leq x \leq 3$. Put a circled dot at the point corresponding to the initial condition, annotated by its coordinates $(1, 0)$. Similarly indicate the point on this curve where $x = 3$ and estimate $y(3)$. [Note that the arrow gridpoint separation is 0.2 in both directions. Confirm this.]
Don't change your estimated value after you later evaluate the solution exactly.

b) Use the linear solution recipe to find the general solution of this differential equation. Simplify it and box it. What is the name of the shape of the curves which make up this family of solutions?

c) Find the solution of this differential equation which satisfies the given initial condition.

...



$x = -2..3, y = -1..2$

d) Evaluate your solution at $x = 3$ numerically to 2 decimal places and mark the corresponding point on your graph with a visible **×** . Is this consistent with your part a) estimated result? Explain.

e) Does your initial value problem solution agree with Maple? If equivalent, show the equivalence. If not, can you find your mistake?

f) Notice that the DE tells us that when $x = 0$, then we must have $y = 0$ (all solutions pass through the origin!) and the general solution shows that they all share the same tangent line, so we have no idea how to continue our solution to the left of the origin by connecting the arrows. Plot the two points on our part c) solution for $x = -1, -2$ (dotted circles!) and draw in the corresponding solution curve which passes through them. [This may take some trial and error to get the curve to pass through both points.]

▶ **solution**

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a) $y=-\frac{1}{4}x$, $\frac{dy}{dx}=-\frac{1}{4}$

$$\underbrace{x\frac{dy}{dx}-2y-\frac{x}{4}=0}_{\text{"separate"}} \rightarrow x(-\tfrac{1}{4})-2(-\tfrac{1}{4}x)-\tfrac{x}{4}=0$$

$$x(-\tfrac{1}{4}+\tfrac{1}{2}-\tfrac{1}{4})=0$$

$$0=0 \checkmark$$

c) $x\frac{dy}{dx}-2y=\frac{x}{4}$ (now divide by x)

$$x^{-2}\left[\frac{dy}{dx}-\frac{2}{x}y=\frac{1}{4}\right] \longrightarrow \frac{d}{dx}(yx^{-2})=\frac{1}{4}x^{-2}$$

$$e^{\int -\frac{2}{x}dx=-2\ln|x|}\quad e^{-2\ln|x|}=(e^{\ln|x|})^{-2}=|x|^{-2}=x^{-2}$$

$$yx^{-2}=\int \tfrac{1}{4}x^{-2}dx$$

$$=\tfrac{1}{4}\left(\frac{x^{-1}}{-1}\right)+C$$

$$=-\frac{1}{4x}+C$$

gen. soln. $\boxed{y=x^2(-\frac{1}{4x}+C) = -\frac{1}{4}x+Cx^2}$

These are $\boxed{\text{parabolas}}$

d) $y(1)=0 \leftrightarrow x=1, y=0$

$0=-\frac{1}{4}(1)+C(1)^2 \rightarrow C=\frac{1}{4}$

$\boxed{y=-\frac{1}{4}x+\frac{1}{4}x^2}=\frac{1}{4}x(x-1)$ (intercepts $x=0,1$ so vertex at $x=1/2$)

e) $y(3)=-\frac{1}{4}(3)+\frac{1}{4}(3)^2=\frac{1}{4}(9-3)=\frac{6}{4}=\frac{3}{2}=\boxed{1.5}$

f) Maple agrees!

g) $y(-1)=-\frac{1}{4}(-1)+\frac{1}{4}(-1)^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}=0.5$

$y(-2)=-\frac{1}{4}(-2)+\frac{1}{4}(-2)^2=\frac{2}{4}+\frac{4}{4}=\frac{3}{2}=1.5$

Note this diagram should have a parabola symmetric about its vertex $x=1/2$, so $y(-2)=y(3)$.