MAT2705-04/05 22s Quiz 2 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

1. $\frac{dy}{dx} = -2xy^2$

initial condition 1: $y(0) = 1$;
initial condition 2: $y(0) = 0$;
initial condition 3: $y(0) = -1$.

a) Indicate these initial data points on the graph by circled dots annotated by their (x, y) values and roughly draw in the corresponding solution curves.

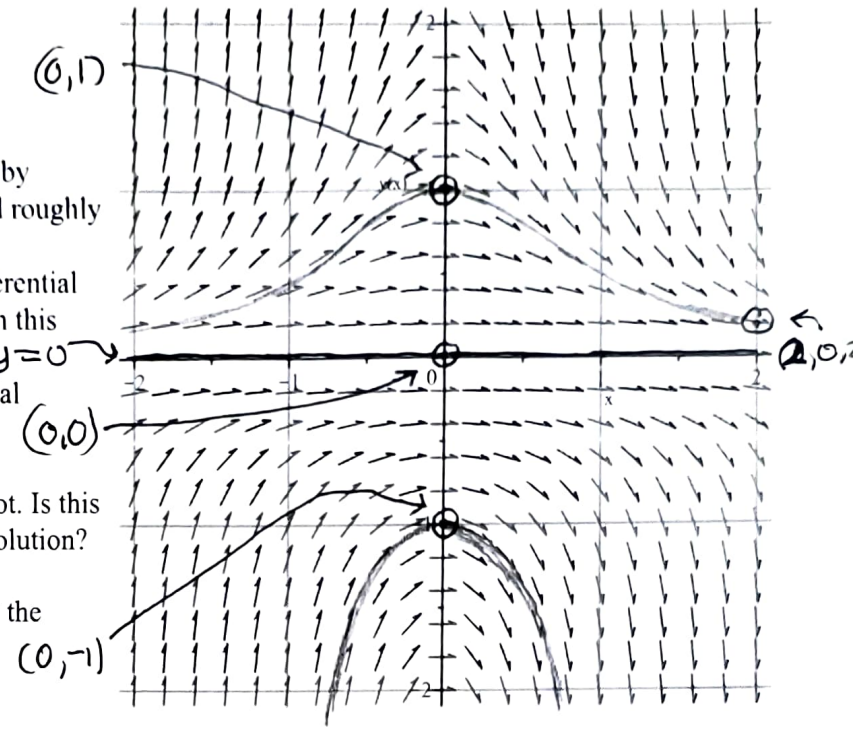
b) Find the (almost) general solution of the differential equation. What obvious solution is missing from this family?

c) Find the solution which satisfies the first initial condition.

d) Evaluate $y(2)$ for this solution and mark the corresponding point on the graph by a circled dot. Is this consistent with your approximate hand drawn solution? Explain.

e) Check by hand that your solution to c) solves the differential equation.

a)



f) Enter the differential equation and its initial condition separated by a comma in Maple. Use the context sensitive menu to solve. Write down the form of the solution that it gives you. Then apply **Simplify, Simplify**. Does it agree with your hand solution? Can you simplify Maple's solution by hand to get the same result? If so, show the steps.

▶ solution

b) $\frac{dy}{dx} = -2xy^2$

$\int y^{-2}\,dy = \int -2x\,dx$

$\frac{y^{-1}}{-1} = -2\left(\frac{x^2}{2}\right) + C_1$

$\frac{1}{y} = x^2 - C_1$

$\boxed{y = \frac{1}{x^2 - C_1}}$

$\boxed{y = 0}$ is also a soln

c) $1 = y(0) = \frac{1}{0 - C_1} = -\frac{1}{C_1}$

$C_1 = -1$

$\boxed{y = \frac{1}{x^2 - (-1)} = \frac{1}{x^2+1}}$

d) $y(2) = \frac{1}{2^2+1} = \frac{1}{4+1} = \frac{1}{5} = \boxed{0.2}$

so the pt $(2, 0.2)$ is on the soln curve.
grid divisions are 0.2.
My hand drawn curve goes right through this point!

e) $y = (x^2+1)^{-1}$

$\frac{dy}{dx} = -(x^2+1)^{-2}(2x+0) = \frac{-2x}{(x^2+1)^2}$

$\frac{dy}{dx} = -2xy^2 \rightarrow \frac{-2x}{(x^2+1)^2} = -2x\left(\frac{1}{x^2+1}\right)^2$

$= \frac{-2x}{(x^2+1)^2}$ ✓

f) $y = -\frac{1}{2}\left(\frac{-2x^2+2}{-1+x^4}\right) = \frac{1}{x^2+1}$ ↑ simplify.

$= \frac{x^2-1}{(x^2-1)(x^2+1)}$

(multiply thru, factor denominator, cancel factors)