MAT2705-04/05 22S Quiz 1 Print Name (Last, First) _____

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use **proper mathematical notation**, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them, unless specifically requested.

1. $\frac{dy}{dx} + y\tan(x) = \cos(x)$, $soln: y = (x + C)\cos(x)$

a) Verify that this y satisfies the given differential equation.

b) Find the solution which satisfies the initial condition $y(\pi) = 0$.

Organize your work as though you were playing professor.

2. a) Write a differential equation that models the situation:
"The rate of change of y with respect to x is inversely proportional to the difference between 1 and y."

b) Explain what sign your constant of proportionality should have so that its solutions for $y < 1$ are increasing functions of x.

Optional. Use Maple to solve this DE. Do you recognize the kind of curve it represents?

▶ solution

① a) $y = (x+C)\cos x$

$y' = (1+0)\cos x + (x+C)(-\sin x)$

$= \cos x - (x+C)\sin x$

$y' + y\tan x = \cos x$

$\cos x - (x+C)\sin x + (x+C)\cos x \tan x \stackrel{?}{=} \cos x$

$\cos x - (x+C)\sin x + (x+C)\sin x \stackrel{?}{=} \cos x$

$\cos x = \cos x$ ✓

b) $y(\pi) = 0 \leftrightarrow x = \pi,\ y = 0$

$0 = (\pi + C)\underbrace{\cos\pi}_{-1}$

$0 = \pi + C$

$C = -\pi$ back sub

$\boxed{y = (x-\pi)\cos x}$

② a) $\frac{dy}{dx} \propto \frac{1}{1-y}$ or $\frac{1}{y-1}$ (2 choices)

$\boxed{\frac{dy}{dx} = \frac{k_1}{1-y} = \frac{k_2}{y-1}}$ (2 choices)

b) $y < 1$: $(0 < 1-y)$ $\quad 1-y > 0 \quad y-1 < 0$

$\underbrace{k_1 > 0}$ $\quad \underbrace{k_2 < 0}$

$\therefore \frac{dy}{dx} > 0 \quad \therefore \frac{dy}{dx} > 0 \leftrightarrow$ increasing function of x

optional Maple

$> y' = \frac{k}{1-y} \rightarrow y = 1 \pm \sqrt{1 - 2kx - 2\_C_1 k}$

So $(y-1)^2 = 1 - 2k(x + \_C) \geq 0$

Isolate x: $2k(x + \_C_1) = 1 - (y-1)^2$

$x = -\_C_1 + \frac{1-(y-1)^2}{2k}$ = quadratic function of y

"parabola" with horizontal symmetry axis

example, $k=1$, $\therefore C_1 = 0$:

$x = \frac{1}{2}(1 - (1-y)^2) = \frac{1}{2}(1 - 1 + 2y - y^2)$

$= y - \frac{1}{2}y^2$