MAT2705-04/05 22S test 2 Answers

① $Ax=0$:

$$\begin{bmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & -4 & -3 & -7 & 0 \\ 2 & -1 & 1 & 7 & 0 \\ 1 & 2 & 3 & 11 & 0 \end{bmatrix}}_{\text{augmented matrix}} \xrightarrow[\text{Maple}]{\text{rref}} \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 5 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 \\ L & L & F & F \end{matrix} \end{array} \rightarrow \begin{aligned} x_1 \quad + x_3 + 5x_4 &= 0 \\ x_2 + x_3 + 3x_4 &= 0 \\ 0 &= 0 \end{aligned}$$

$x_3 = t_1,\ x_4 = t_2$

$$\downarrow$$

$$x_1 = -t_1 - 5t_2$$
$$x_2 = -t_1 - 3t_2$$

solution

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t_1 - 5t_2 \\ -t_1 - 3t_2 \\ t_1 \\ t_2 \end{bmatrix}}$$

b) $$= t_1 \underbrace{\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}}_{u_1} + t_2 \underbrace{\begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}}_{u_2}$$

$\{u_1, u_2\}$ is a basis of the sdn space

c) $$\boxed{\begin{aligned} u_1:&\ -v_1 - v_2 + v_3 = 0 \\ u_2:&\ -5v_1 - 3v_2 + v_4 = 0 \end{aligned}}$$

② $$x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -7 \\ 7 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & -3 \\ 2 & -1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 7 \\ 11 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -4 & -3 \\ 2 & -1 & 1 \\ 1 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -7 \\ 7 \\ 11 \end{bmatrix} \underset{\text{Maple}}{=} \frac{1}{7} \begin{bmatrix} -6 & 10 & -7 \\ -7 & 7 & -7 \\ 5 & -6 & 7 \end{bmatrix} \begin{bmatrix} -7 \\ 7 \\ 11 \end{bmatrix}$$

$$= \frac{1}{7} \begin{bmatrix} (-6)(-7) + 10(7) + (-7)(11) \\ -7(-7) + 7(7) - 7(11) \\ 5(-7) - 6(7) + 7(11) \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 42 + 70 - 77 \\ 49 + 49 - 77 \\ -35 - 42 + 77 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 35 \\ 21 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$$

Thus $$\boxed{\begin{bmatrix} -7 \\ 7 \\ 11 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}}$$

b) check $$\left(= \begin{bmatrix} 5 - 12 \\ 10 - 3 \\ 5 + 6 \end{bmatrix} = \begin{bmatrix} -7 \\ 7 \\ 11 \end{bmatrix} \right)$$

(v_3 not needed)

③ b) $$y_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y_2 \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -8 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

so $$\boxed{\begin{bmatrix} 0 \\ -8 \end{bmatrix} = -2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} -2 \\ 3 \end{bmatrix}}$$

b) $$\stackrel{?}{=} \begin{bmatrix} -4 + 4 \\ -2 - 6 \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \end{bmatrix} \checkmark$$ agrees

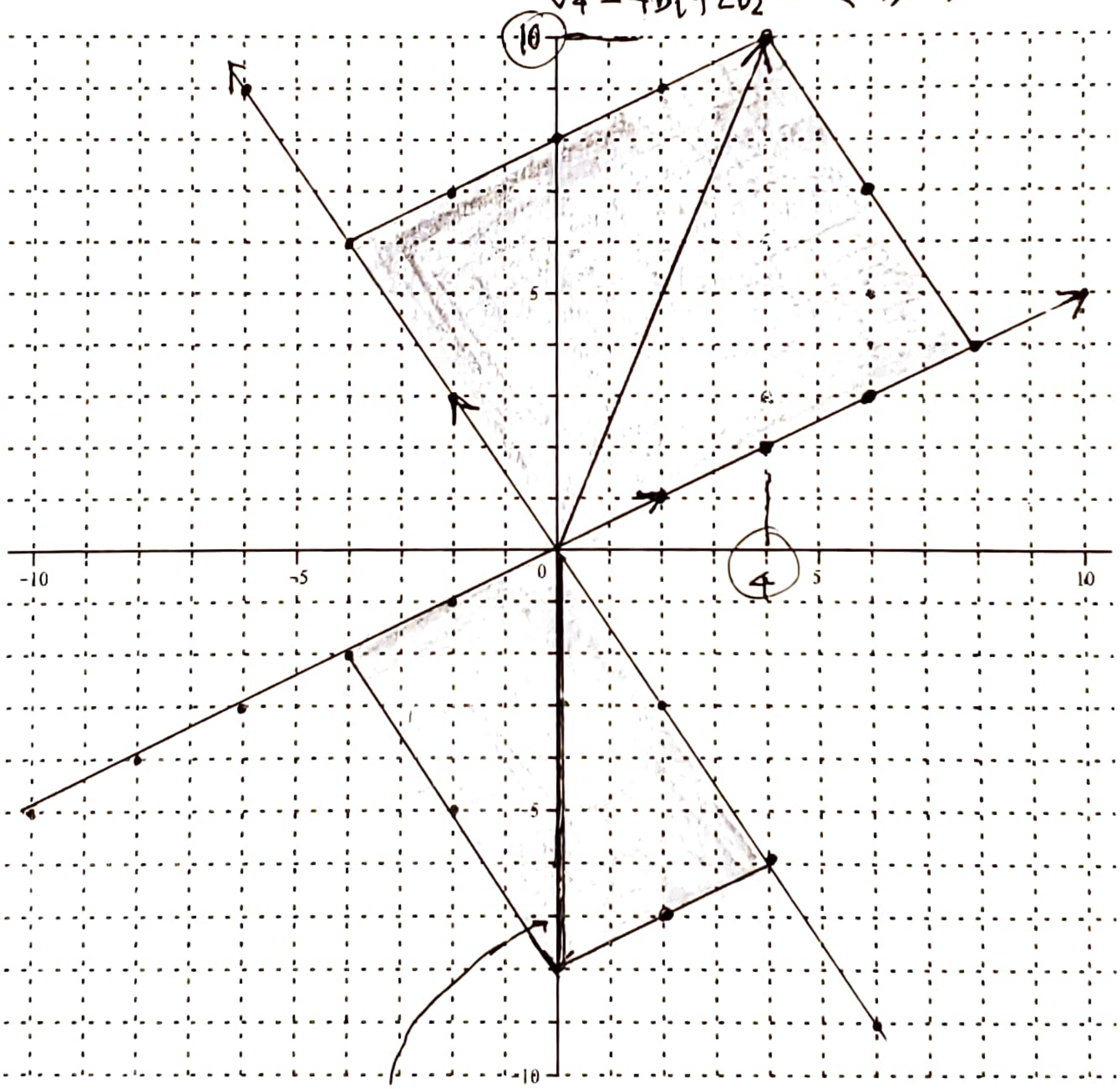
c) $v_4 = 4b_1 + 2b_2 = 4\langle 2, 1\rangle + 2\langle -2, 3\rangle = \langle 8-4, 4+6\rangle = \langle 4, 10\rangle$ agrees with plot ✓

3. a) On the grid below, **draw in** arrows representing the new basis vectors $\vec{b}_1 = \langle 2, 1\rangle$ and $\vec{b}_2 = \langle -2, 3\rangle$ and $\vec{v}_3 = \langle 0, -8\rangle$ and **label** them by their symbols. **Extend** the basis vectors $\{\vec{b}_1, \vec{b}_2\}$ to the corresponding coordinate axes for (y_1, y_2) and **mark** the positive direction with an arrow head and the axis label. Mark off tickmarks on these axes for integer values of the new coordinates. Then **draw in** the parallelogram with edges parallel to the new axes for which $\vec{v}_3$ is the main diagonal and shade it in in pencil lightly. Read off the coordinates (y_1, y_2) of $\vec{v}_3$ with respect to these two vectors (write them down) and **express** $\vec{v}_3$ as a linear combination of these vectors; **put this equation** at the tip of this vector: $\vec{v}_3 = (\ldots)\,\vec{b}_1 + (\ldots)\,\vec{b}_2$

b) Now use the inverse basis changing matrix to express $\vec{v}_3$ as a linear combination of the two two vectors (show all steps in this process, including the details of matrix multiplication and arithmetic), box it and then check your linear combination by expanding it out. Does your matrix result agree with your graphical result in part a)?

c) **Draw in** the arrow representing the vector $\vec{v}_4$ whose new coordinates are $(y_1, y_2) = (4, 2)$ and **label** the tip of $\vec{v}_4$ by its symbol. Draw in the projection parallelogram associated with the new coordinates and lightly shade it in in pencil. Determine its old coordinates (x_1, x_2) graphically. Then evaluate them using a linear combination or matrix product.

$V_4 = 4b_1 + 2b_2 = \langle 4, 10\rangle$



$V_3 = -2b_1 - 2b_2 = \langle 0, -8\rangle$